

## Presentation Slides

to be used in conjunction with the Developing Spatial Thinking Curriculum.

- Module 1 – Solids of Revolution
- Module 2 – Combining Solids
- Module 3 – Isometric Sketching
- Module 4 – Orthographic Projection
- Module 5 – Incline and Curved Surfaces
- Module 6 – Flat Patterns
- Module 7 – Rotation of Objects about 1 Axis
- Module 8 – Rotation of Objects about 2 Axes
- Module 9 – Object Reflections and Symmetry
- Module 10 – Cutting Planes

Accessible at

<http://www.higheredservices.org/spatial-course-materials/>

# Surfaces and Solids of Revolution

## Module 1

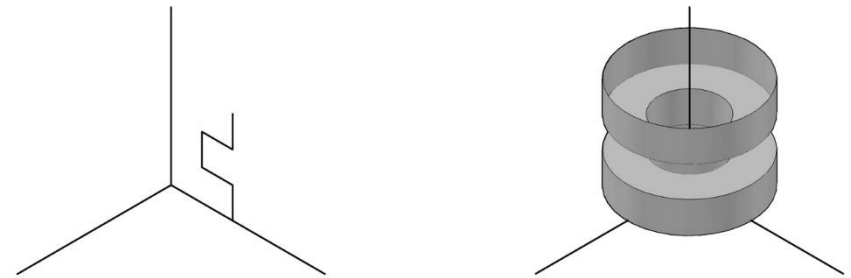
# Session Topics

- Surfaces and Solids of Revolution
- Degree of Revolution
- Hollow Objects
- Visualizing Revolution

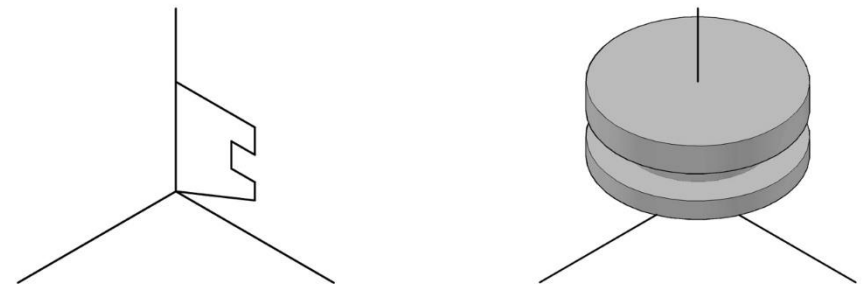
# Surfaces and Solids of Revolution

Surfaces and Solids of Revolution are formed when a 2-D shape is revolved about an axis

- Surfaces result if the shape is open
- Solids result if the shape is closed



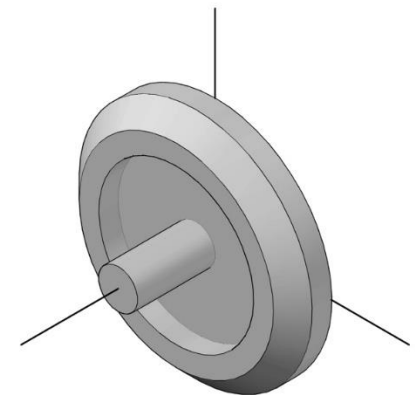
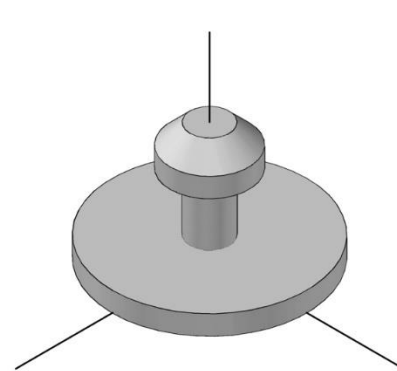
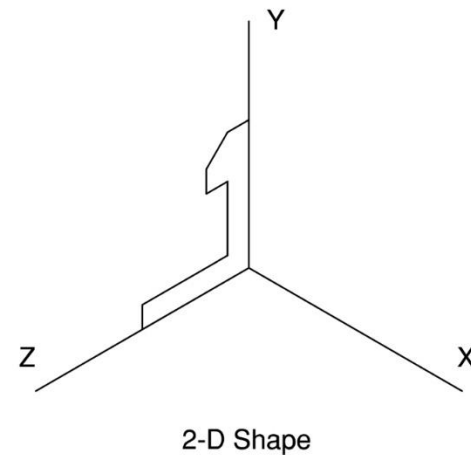
2-D Shape and Surface of Revolution



Closed 2-D Shape and Solid of Revolution

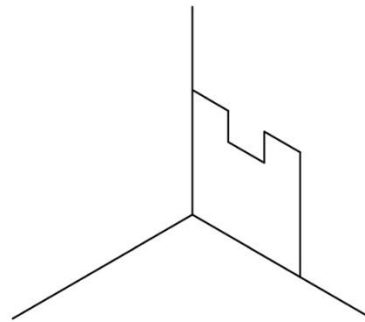
# Solids of Revolution

The resulting 3-D object depends on the axis about which the 2-D shape was revolved

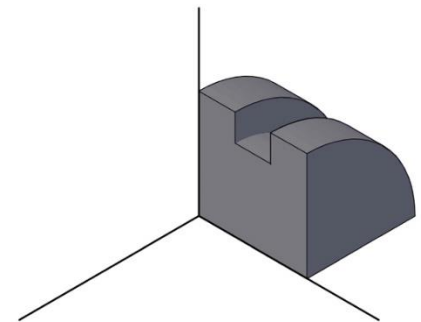


# Angle of Revolution

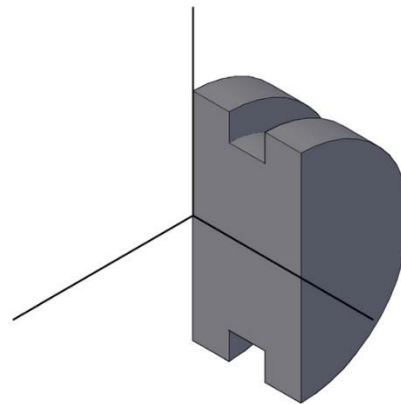
Resulting 3-D object also depends on the degree of angular revolution



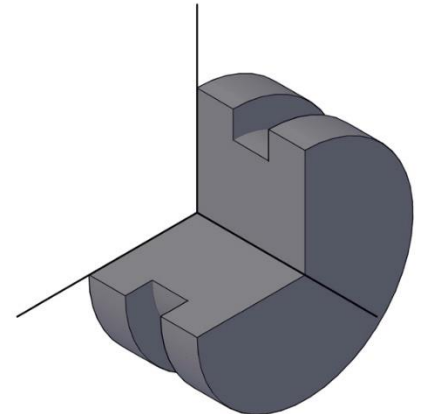
2-D Shape



Shape Revolved 90 Degrees



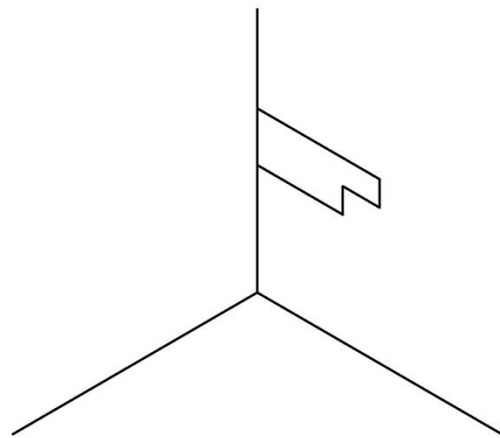
Shape Revolved 180 Degrees



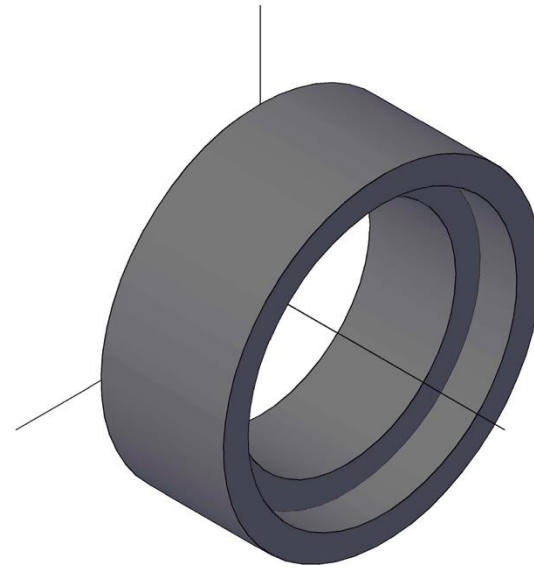
Shape Revolved 270 Degrees

# Hollow Objects

If a 2-D shape is located "away from" the axis of revolution, a solid of revolution with a cylindrical hole in it will result



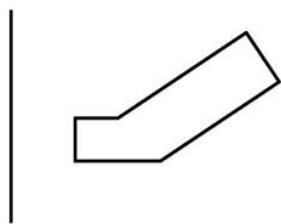
2-D Shape Located Away  
From Axis of Revolution



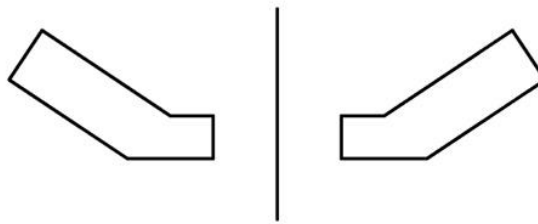
"Hollow" 3-D Part

# Visualizing Revolutions

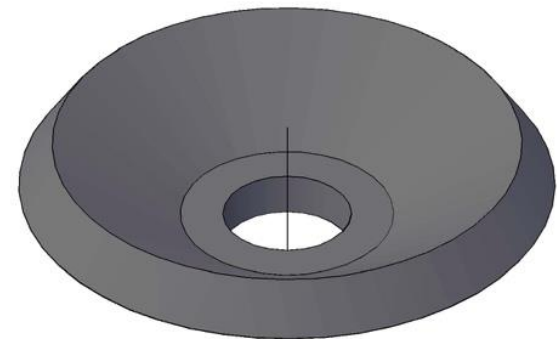
- To visualize a revolution, first think about mirroring the shape about the axis of revolution and then forming a cylindrically-shaped object from the two shapes



2-D Shape



2-D Shape Mirrored About  
Axis of Revolution



2-D Shape Mirrored About  
Axis of Revolution



# Course Software...

- Work through the surfaces and solids of revolution software module

# Surfaces and Solids of Revolution Homework...

- Complete the following pages in Module 1
- rev-
  - 1 - 2
  - 5 - 7
  - 9 - 11
  - 13 - 15

# Combining Solids

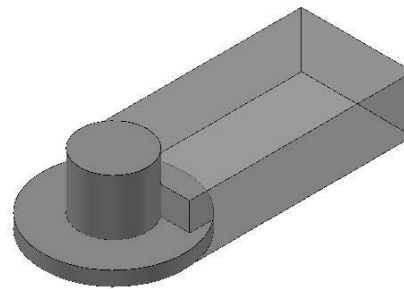
## Module 2

# Session Topics

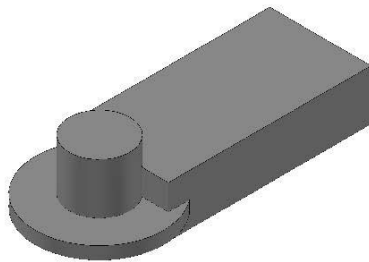
- Combining Overlapping Objects
- Volume of Interference
  - Cutting
  - Joining
  - Intersecting
- Multiple Combinations
- Visualizing Combinations

# Combining Solids

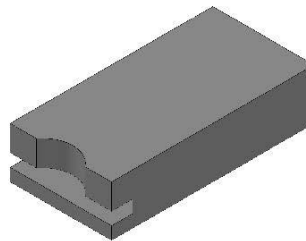
- Two overlapping objects can be combined by cutting, joining, or intersecting



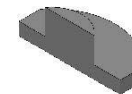
Original Two Objects



Objects Joined



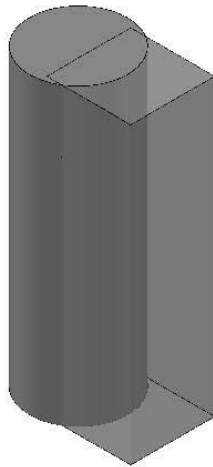
Objects Cut



Objects Intersected

# Volume of Interference

- The Volume of Interference is defined as the overlapping volume between two objects



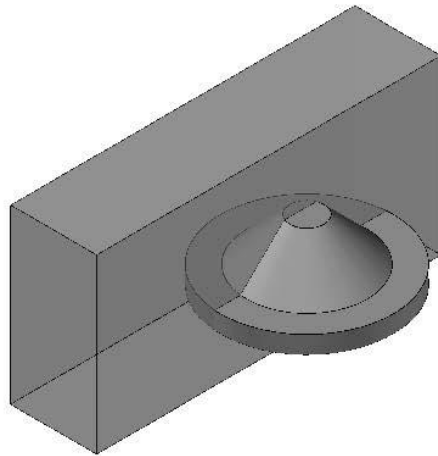
Two Overlapping Objects



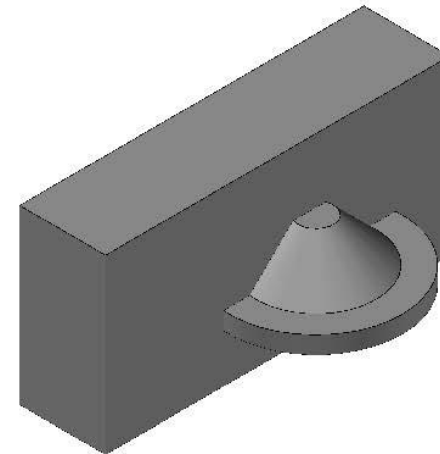
Volume of Interference

# Joining Objects

- When two objects are joined, the volume of interference is absorbed into the final object



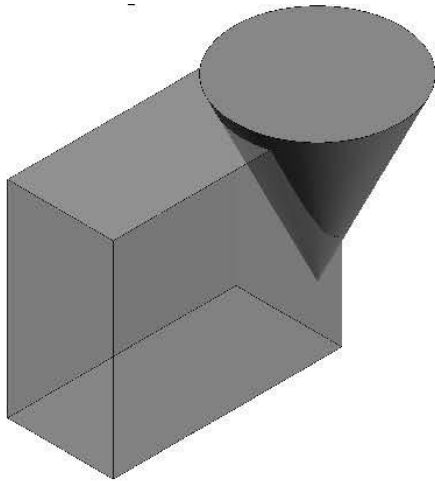
Two Overlapping Objects



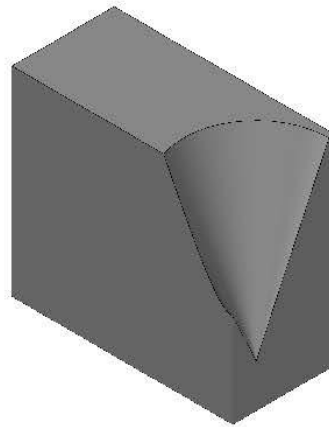
Objects Joined

# Cutting Objects

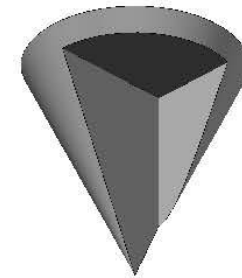
- When two objects are cut, the volume of interference is removed from the object being cut.



Overlapping Objects



Cone Cuts Block

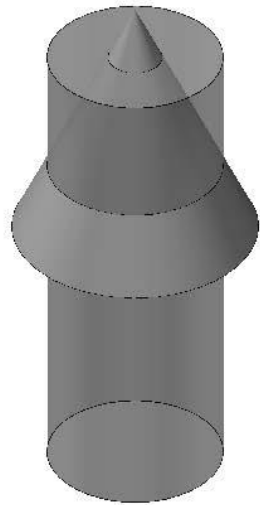


Block Cuts Cone

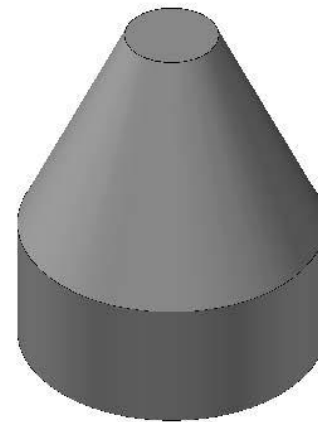


# Intersecting Objects

- When two objects are intersected, the volume of interference becomes the new object.



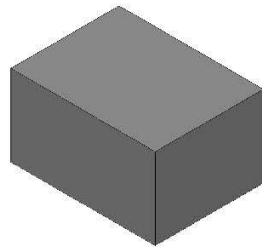
Two Overlapping Objects



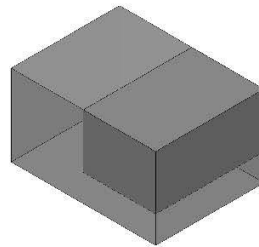
Intersected Objects

# Multiple Combinations

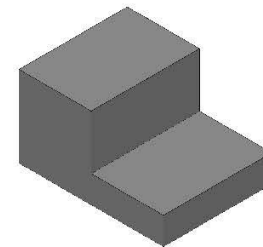
- Complicated objects can be created through cutting, joining, or intersecting parts.



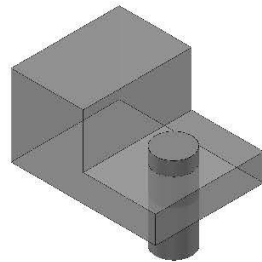
Create Block



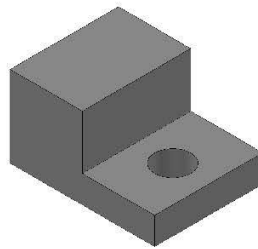
Create Small Block



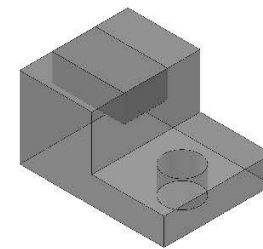
Small Block Cuts Larger Block



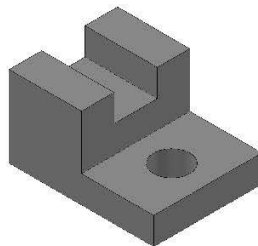
Create Cylinder



Cylinder Cuts Object



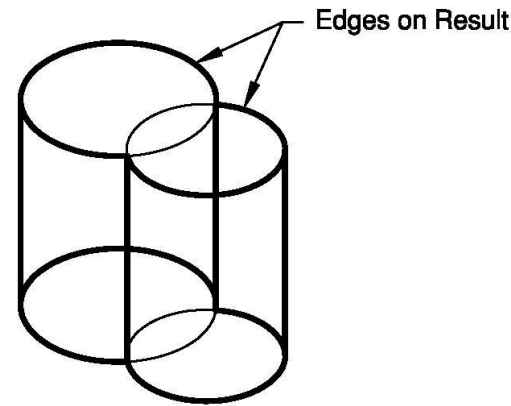
Create Small Block



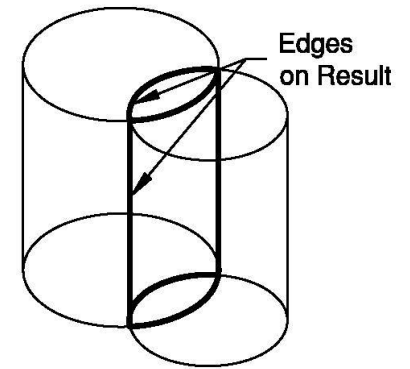
Small Block Cuts Object -- Desired Result

# Visualizing Combinations

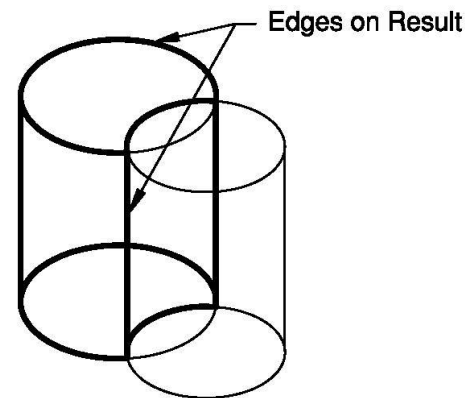
- When presented with two overlapping objects, try to visualize which edges will remain after a combining operation has been performed



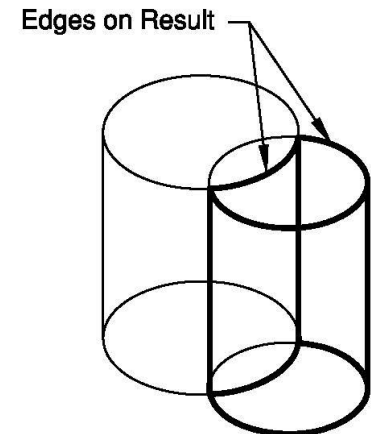
Cylinders Joined



Cylinders Intersected



Small Cylinder  
Cuts Large Cylinder



Large Cylinder Cuts  
Small Cylinder

# Course software ...

- Work through the combining solids software module

# Combining Solid Objects

## Homework ...

- Complete the following pages in Module 2
- ob/int-
  - 2 - 3
  - 6 - 7
  - 9 – 11
  - 13-14
  - Problem 1 on page 2 is not a typo – the resulting object is nothing
  - for pages 13 & 14 show only visible lines

# Isometric Drawings & Coded Plans

## Module 3

# Session Topics

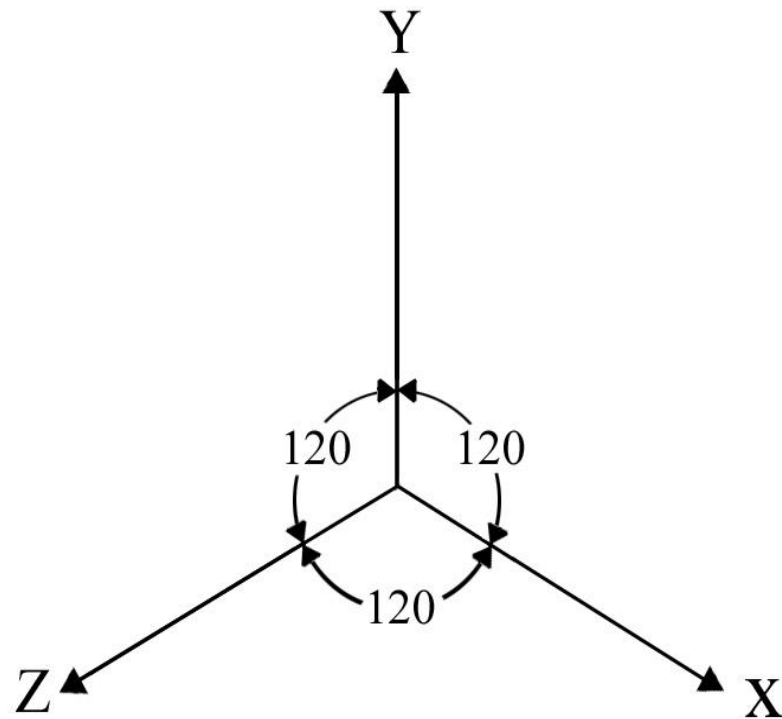
- Isometric Axes
- Coded Plans
- Objects from Multiple Viewpoints
- Isometric Sketching

# Isometric Sketching

- Used to portray a 3-D object on a 2-D sheet of paper.
- The relationship between the 3-D axes appears differently when projected onto a 2-D surface, i.e., perpendicular angles no longer appear perpendicular.

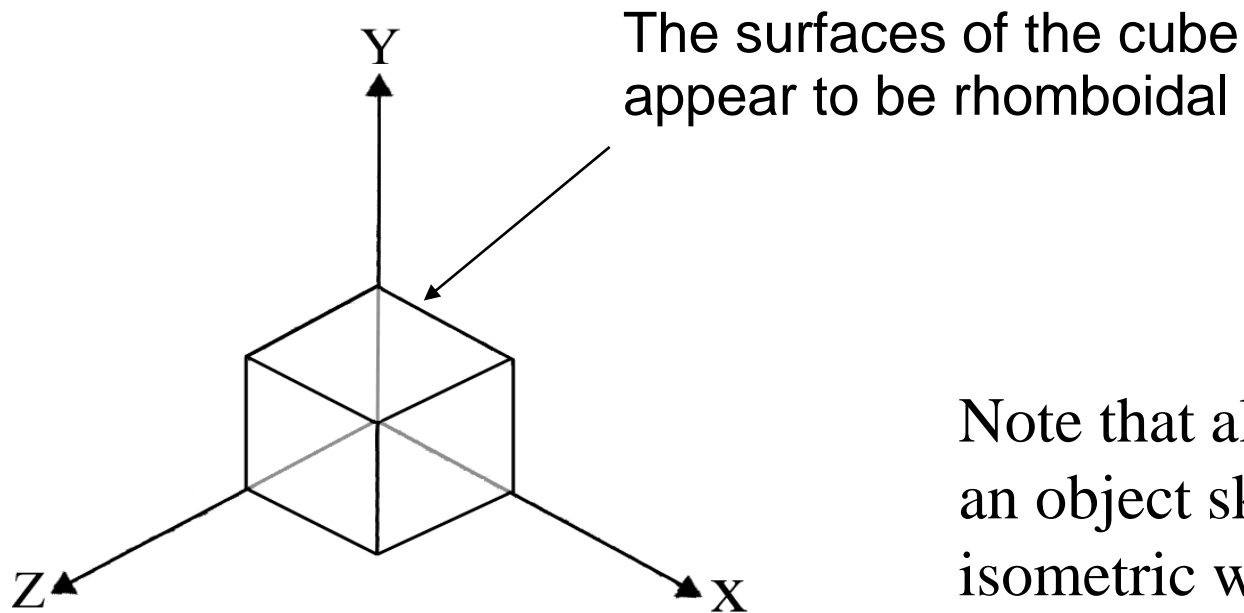


# Isometric Axes



# Isometric Sketch of Cube

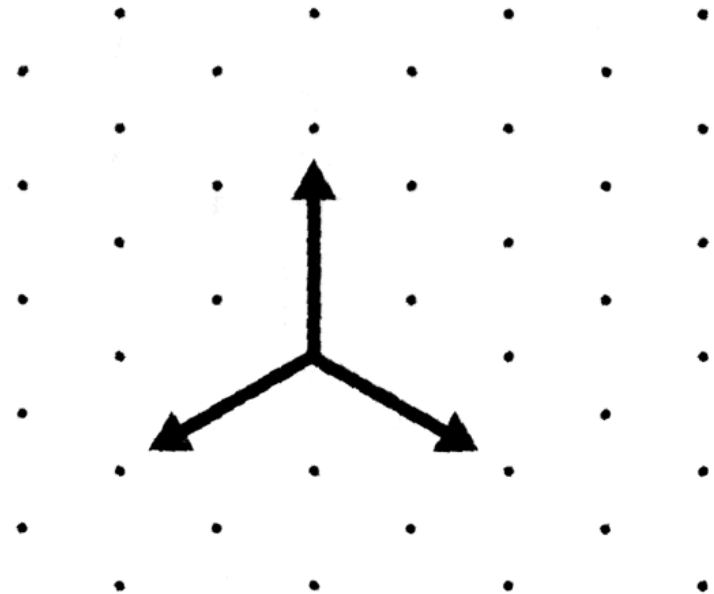
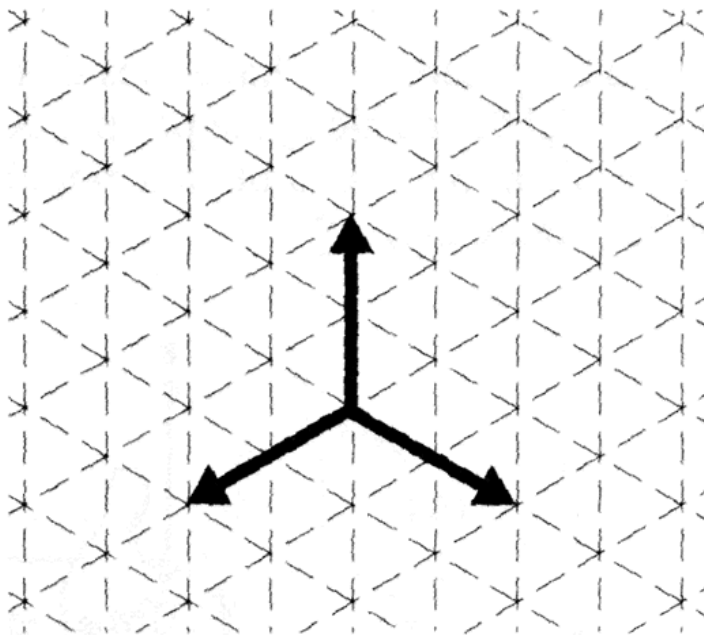
- Isometric sketches are made as if you were looking down a diagonal of a cube



Note that all surfaces of an object sketched in isometric will appear distorted!

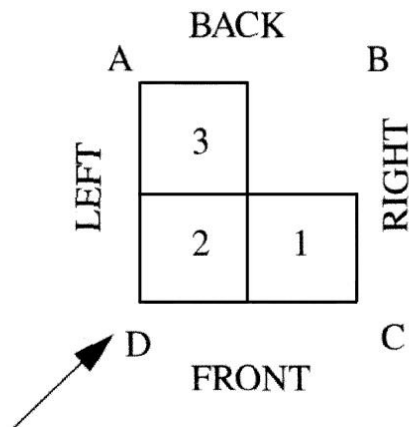
# Isometric Axes

- Isometric grid paper or dot paper is a useful tool for constructing isometric drawings.

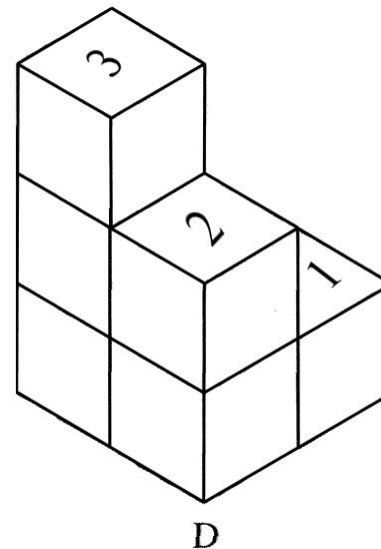


# Coded Plans

- Pictorial sketches can be made from coded plans.
- Coded plans define the shape of a building made of blocks.



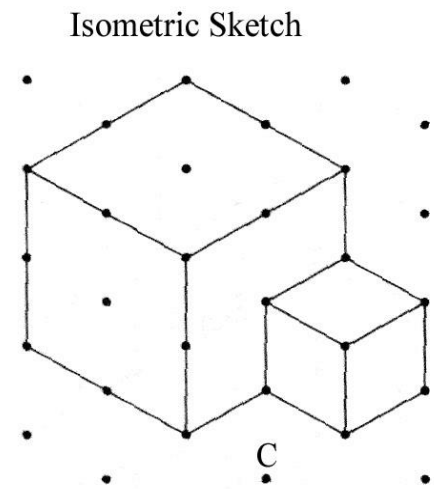
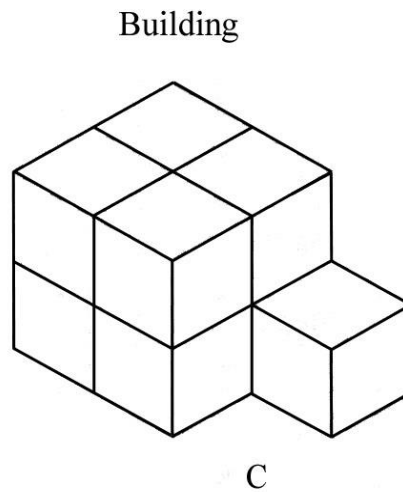
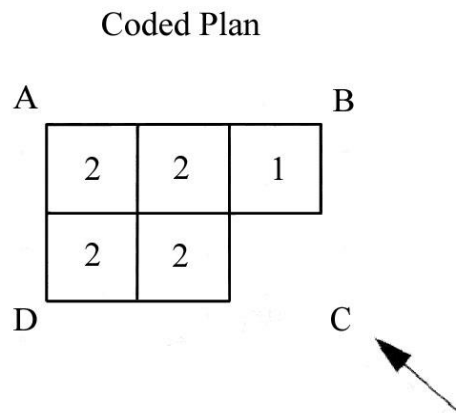
Coded Plan



Building

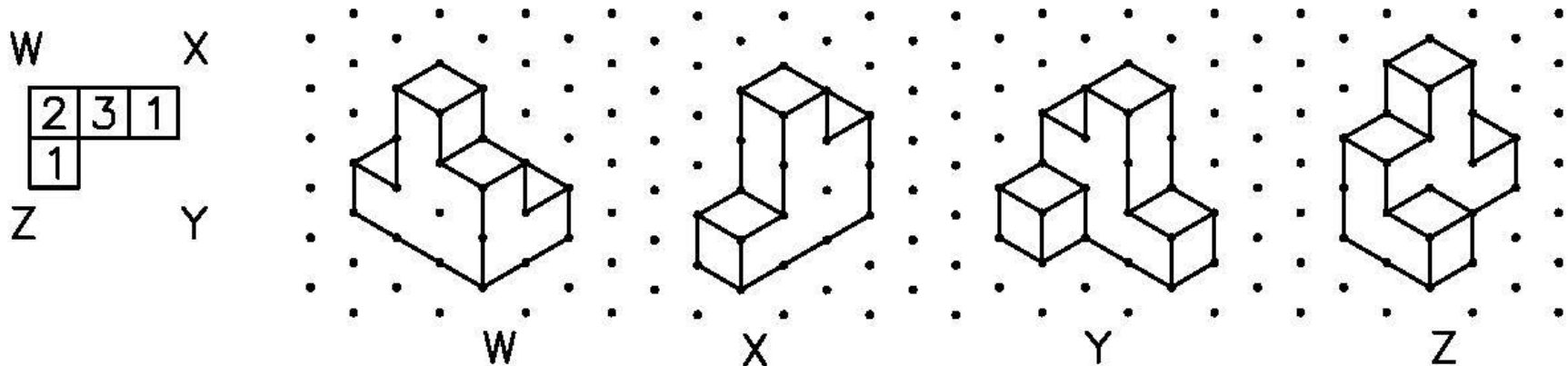
# Coded Plans

- Isometric sketch from corner C.



# Different Viewpoints

- The object will appear differently depending on the corner you are viewing it from



# Isometric Sketching

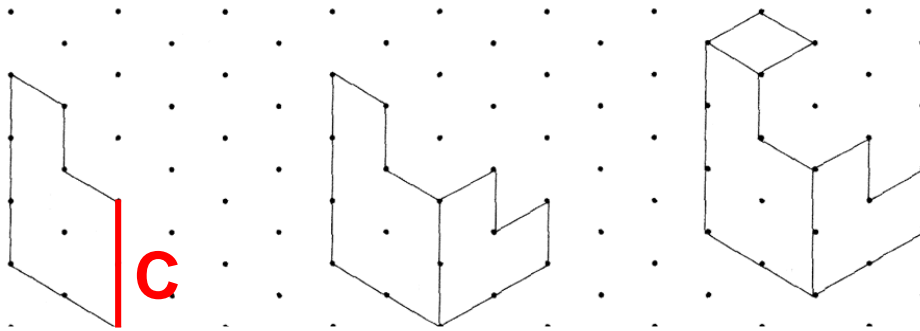
3	1
2	1

**C**

*Guidelines for constructing isometric drawings:*

1. Draw edge "C".
2. Sketch surface to the right or left of edge "C".
3. Draw a surface that shares an edge with the surface just drawn.
4. Continue drawing one surface at a time until the object is complete.

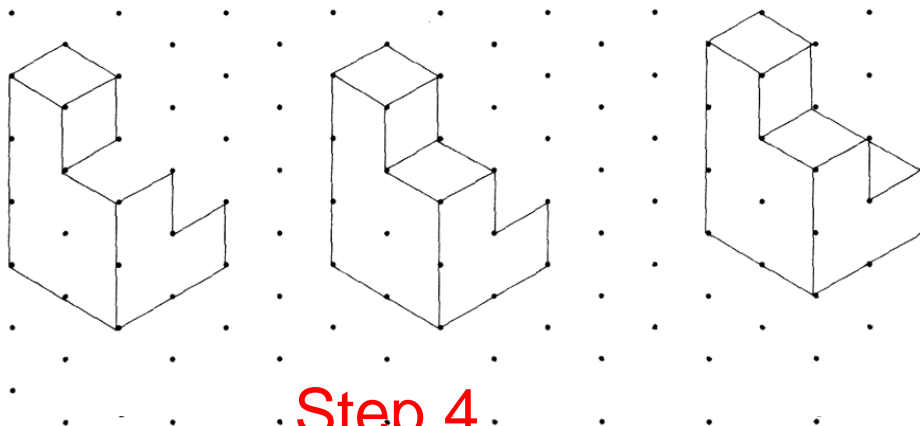
NOTE: Do not show the individual blocks in the building. Just show edges where surfaces intersect.



Steps 1 & 2

Step 3

Step 4 ...



Step 4 ...

# Course Software.....

- Work through the Isometric Drawings and Coded Plans software module



# Homework

- Complete the following pages in Module 3
- iso-
  - 1 - 5
  - 8 - 11

# Orthographic Drawings

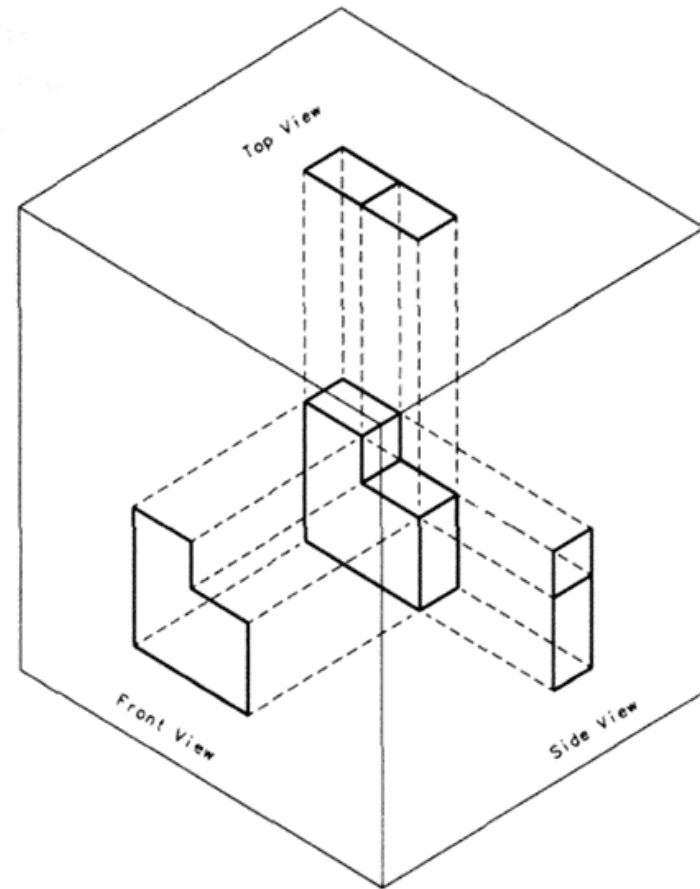
## Module 4

# Session Topics

- Orthographic Projection
- Normal Surfaces
- Hidden Lines
- Isometric Sketching from Orthographic Projections

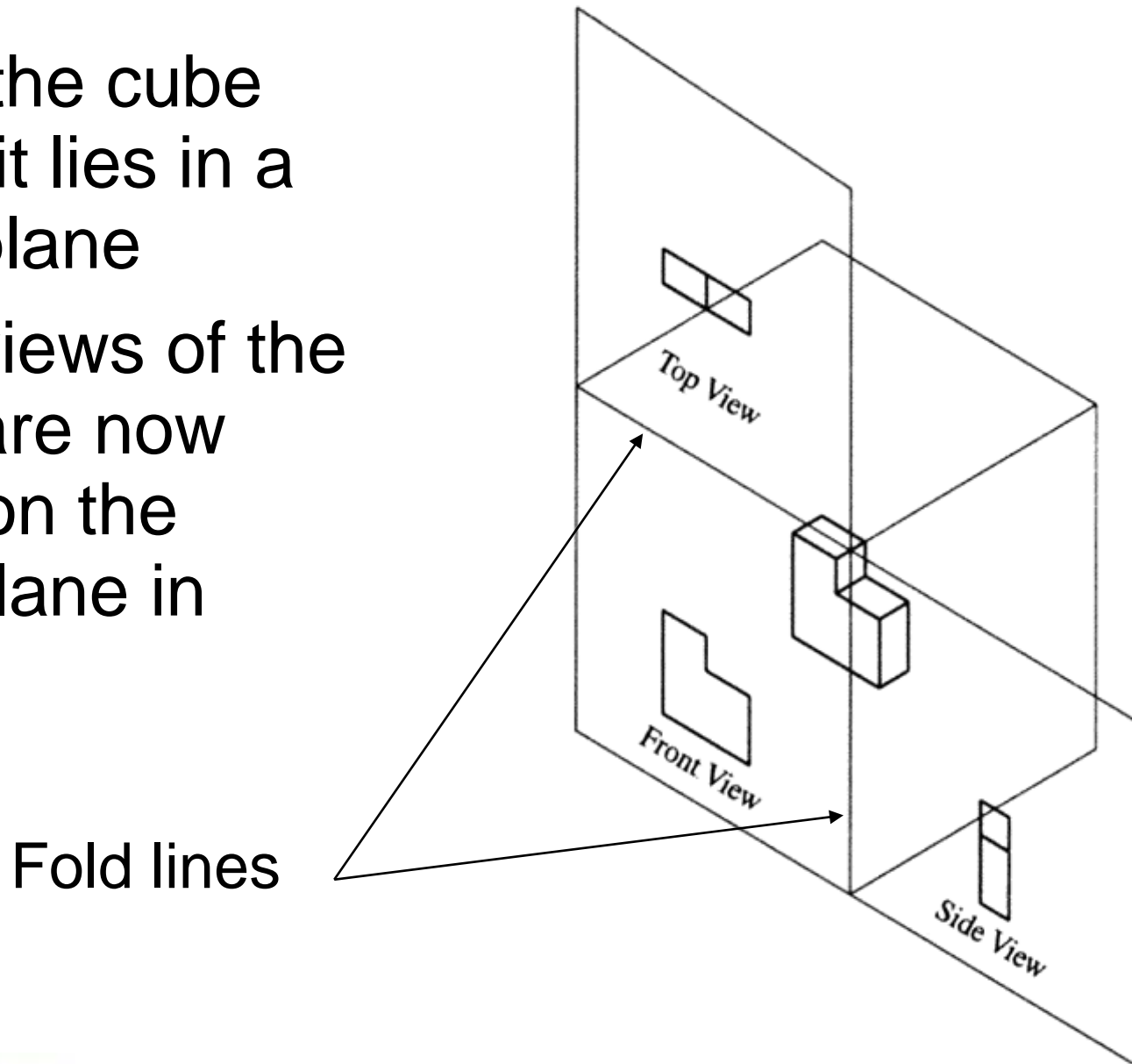
# Orthographic Projection

- Imagine an object is surrounded by a glass cube.
- The object's surfaces are projected onto the faces of the glass cube.
- The projection rays are perpendicular to the panes of glass



# Orthographic Projection

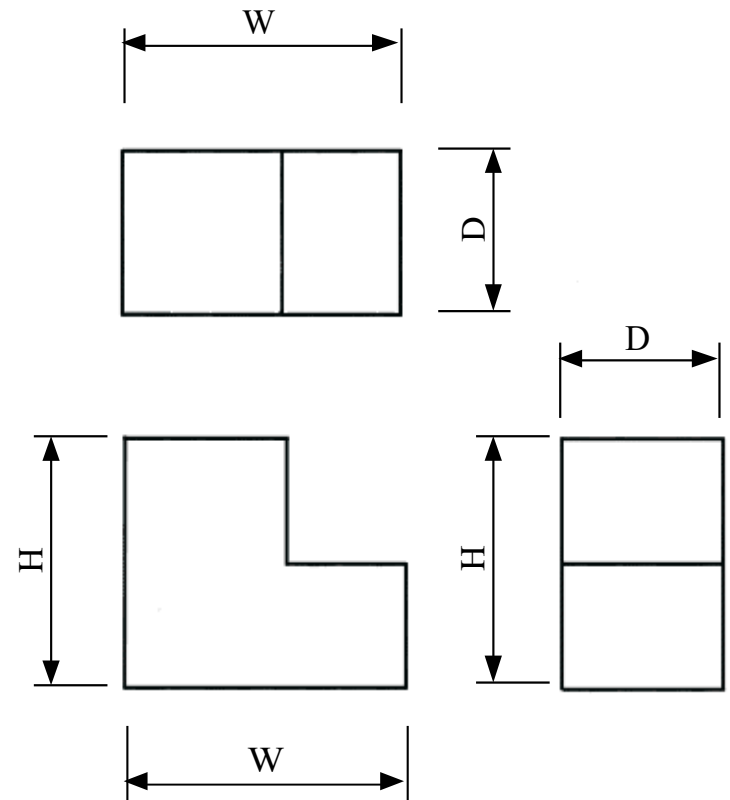
- Unfold the cube so that it lies in a single plane
- Three views of the object are now visible on the same plane in space



# Orthographic Projection

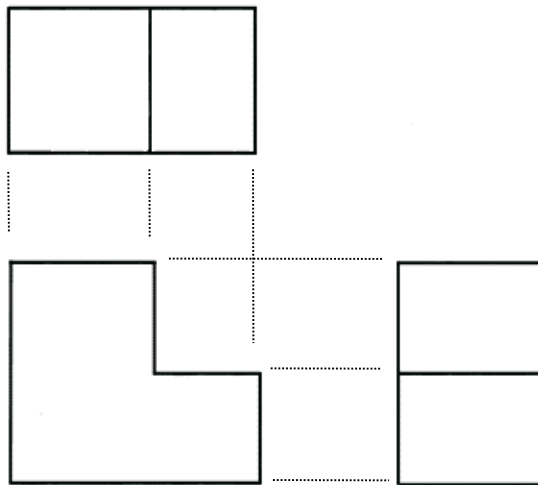
When the glass cube is unfolded each view shows two dimensions of the object:

- Front view: Height and Width
- Top view: Width and Depth
- Right view: Depth and Height

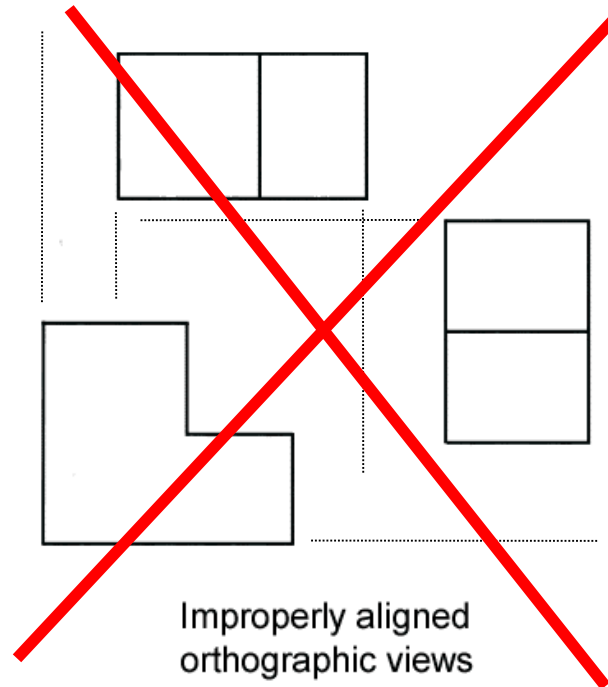


# Orthographic Projection

Views are aligned with one another (features project from one view to the next)

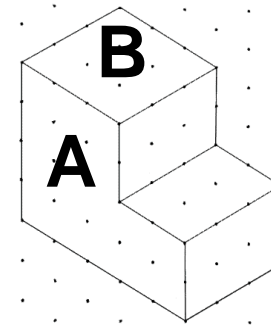


Properly aligned  
orthographic views

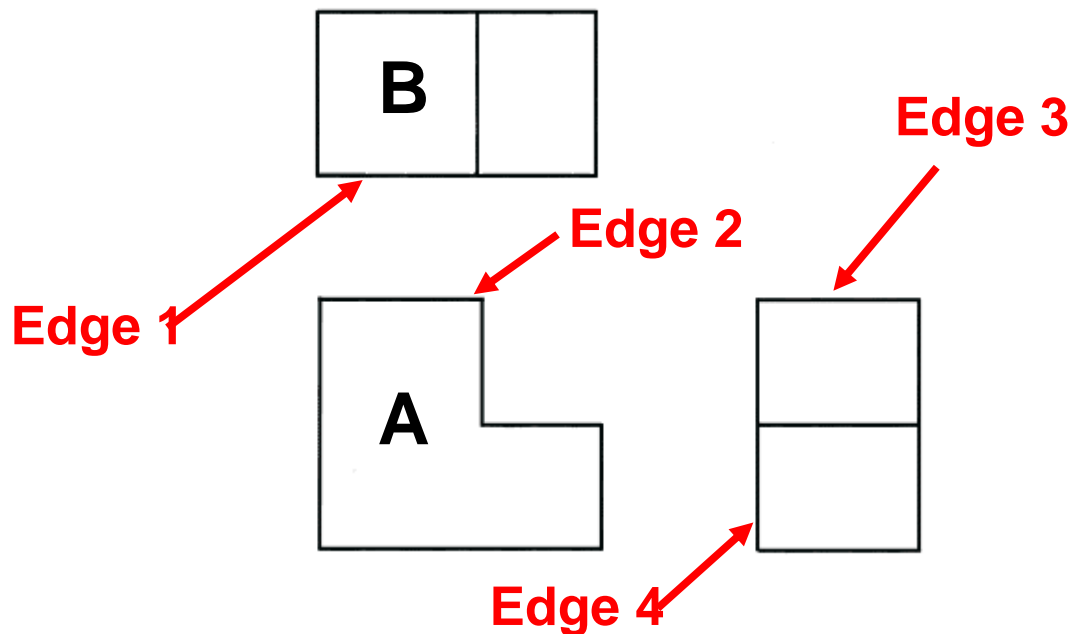


Improperly aligned  
orthographic views

# Normal Surfaces



- Parallel to one of the six glass panes of our transparent cube
- Perpendicular (normal) to the projectors to/from that plane



Shown true size and shape in the view that they are parallel to  
Seen as edges (lines) in the other principal views

Surface A appears as edge 1 and 4 in top & right views

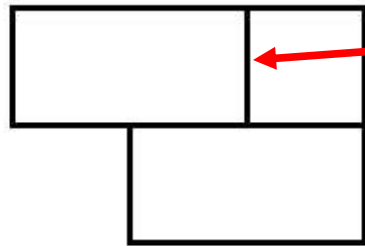
Surface B appears as edges 2 and 3



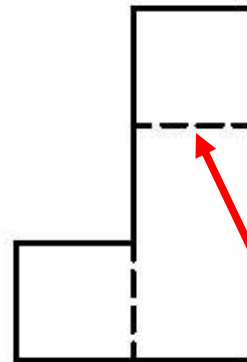
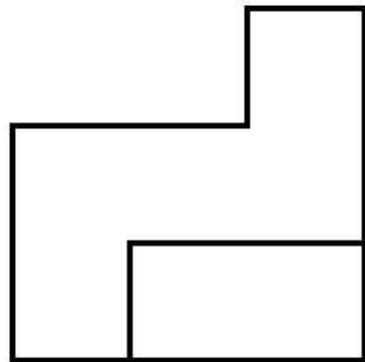
# Hidden Lines

- Some object have edges which cannot be seen from certain viewing angles
- Showing these edges provides valuable graphical information
- Visible edges are continuous (solid) lines (*object lines*)
- Hidden edges are dashed lines (*hidden lines*) to avoid confusing them with visible edges

# Hidden Lines



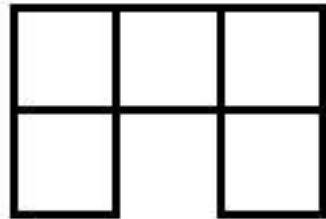
**Visible Line (Solid lines, can be seen in a given view, aka: Object Lines)**



**Hidden Line (Dashed lines; can't be “seen” in that view)**

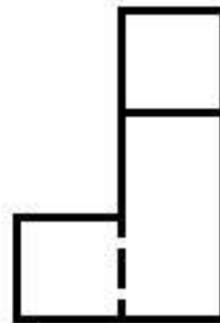
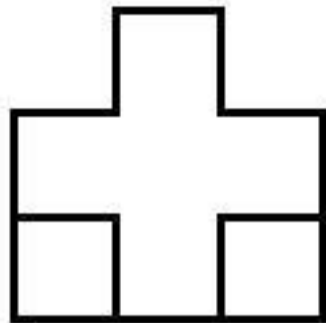
# Hidden Lines and Object Lines Together

- If a solid line and a hidden line are on top of each other, show only the solid line



2	3	2
1		1

Coded Plan

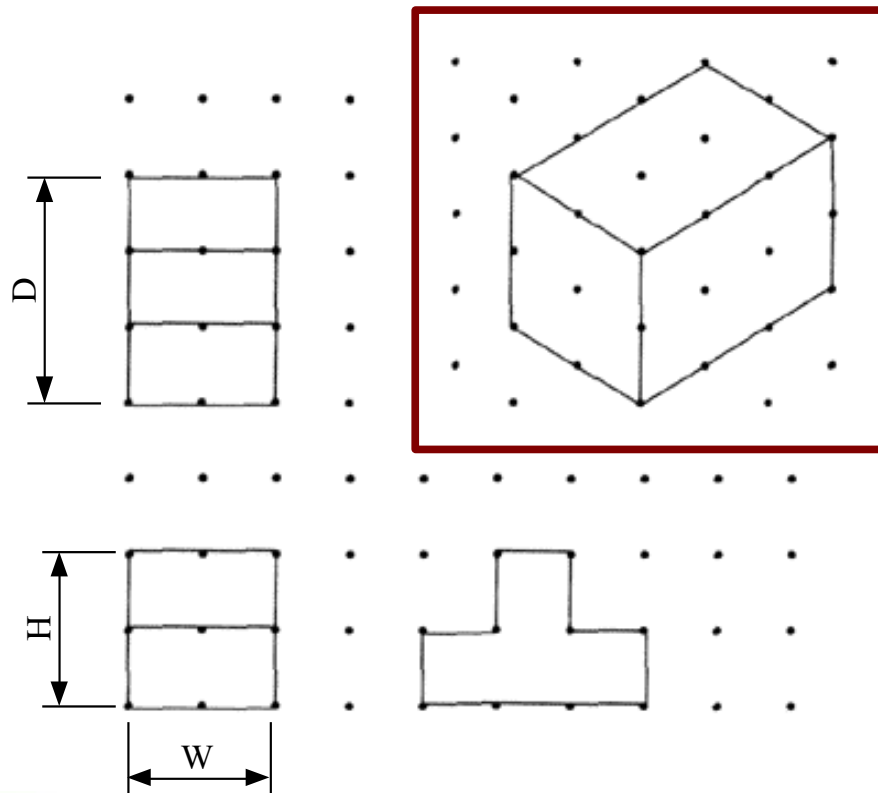


# Isometric Sketches from Orthographic Views

- Sometimes you are asked to construct Isometric sketches from Orthographic views to develop visualization skills.
  - The box method is one way to do this.
  - For some problems, the box method may not be very helpful.

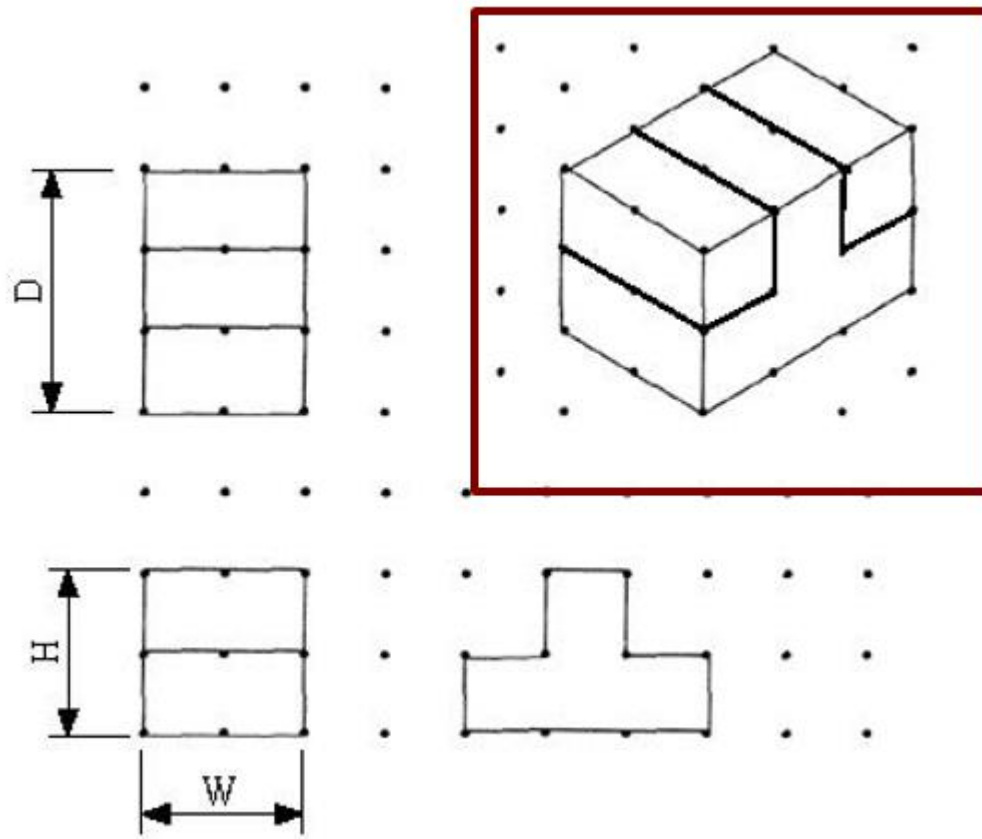
# Isometric Sketches from Orthographic Views

1. Find the object's overall dimensions from the orthographic views and sketch that size box on isometric dot paper.



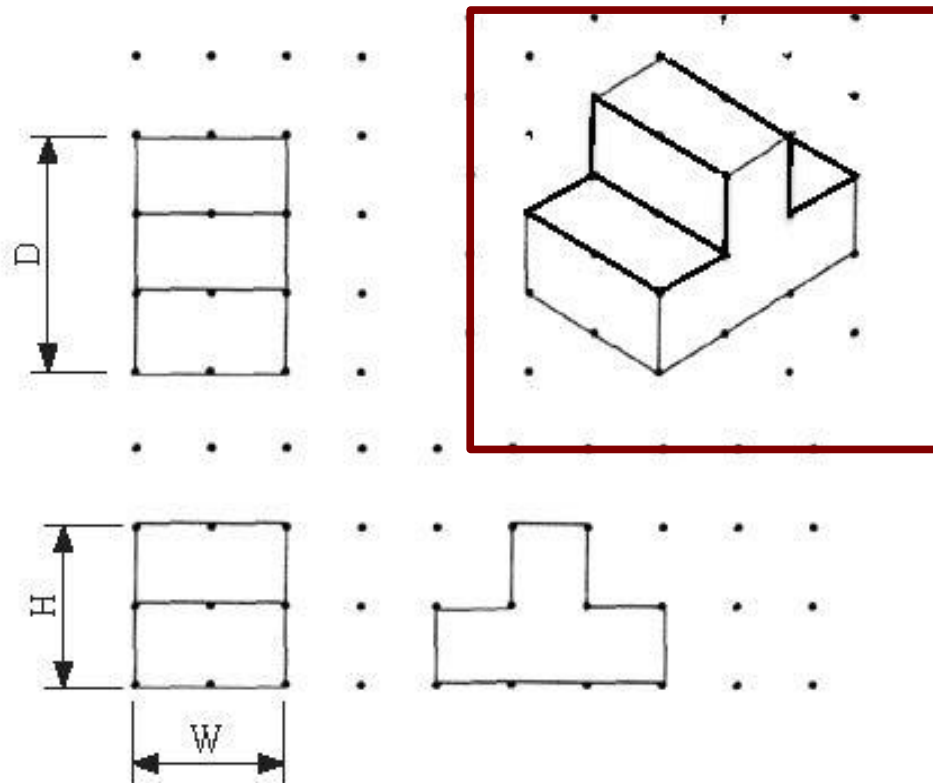
# Isometric Sketches from Orthographic Views

2. Sketch the top, front, and right side views in their appropriate locations on the box.



# Isometric Sketches from Orthographic Views

3. Add/remove lines until the view is complete.



# Course Software.....

- Complete the software module on Orthographic Projections



# Orthographic Drawings

## Homework ...

- Complete the following pages in Module 4
- ortho-
  - 1 - 2
  - 5 - 8
  - 11 - 12
  - 15 - 18
  - 23 - 24

# Inclined and Curved Surfaces

## Module 5

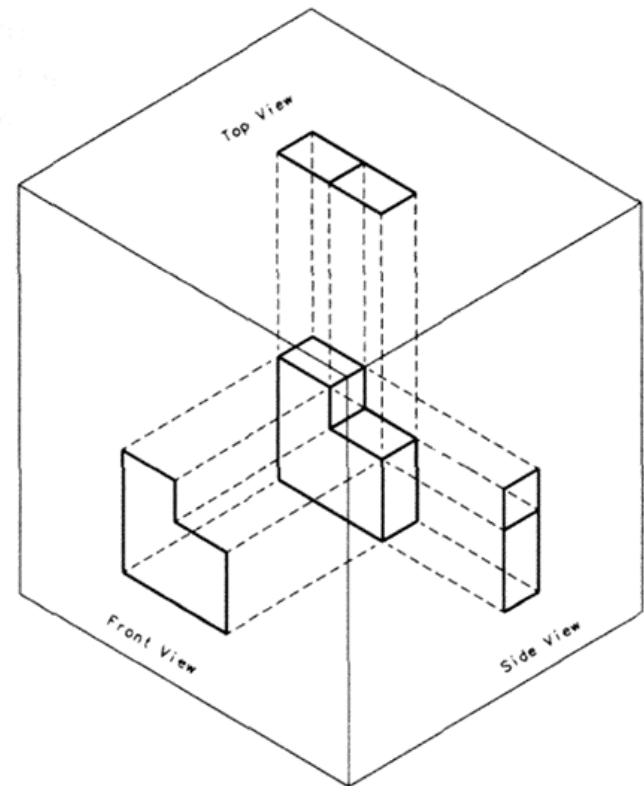
# Session Topics

- Orthographic projections of inclined and single-curved surfaces
- Drawing isometric sketches from orthographic views of objects with inclined and single-curved surfaces

Review:

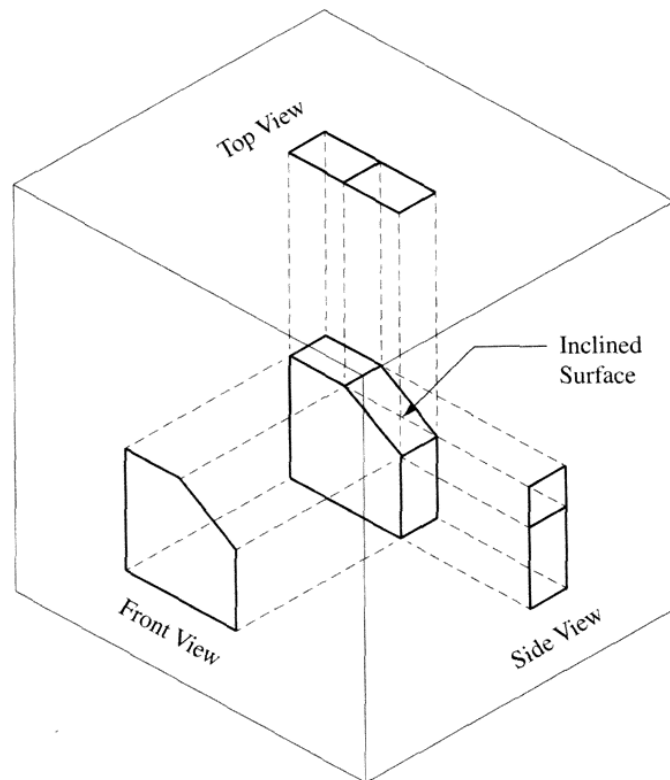
## Orthographic Projection: Normal Surfaces

- Normal surfaces appear as a surface in one view, an edge in the other two views.
- Normal surfaces are shown true size and shape in the view they are parallel to.



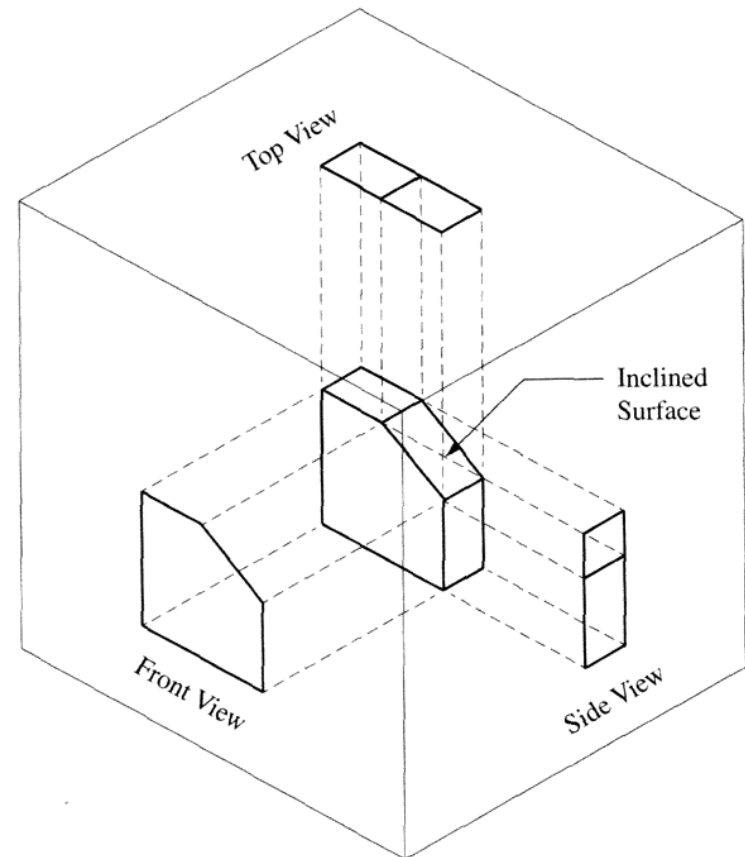
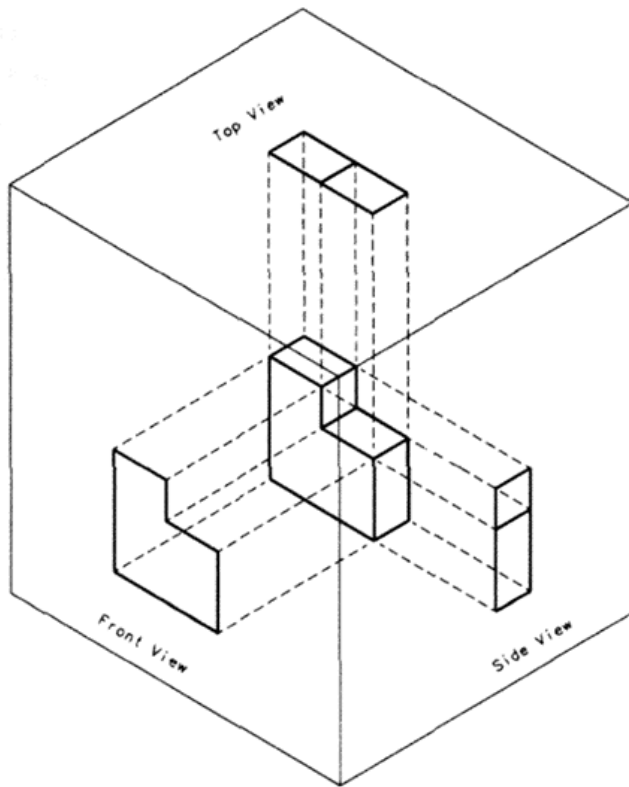
# Inclined Surfaces

- Are not parallel to any of the principal views
- Are perpendicular to one of the three views
- Appear as an area in two views, as an edge in the third view



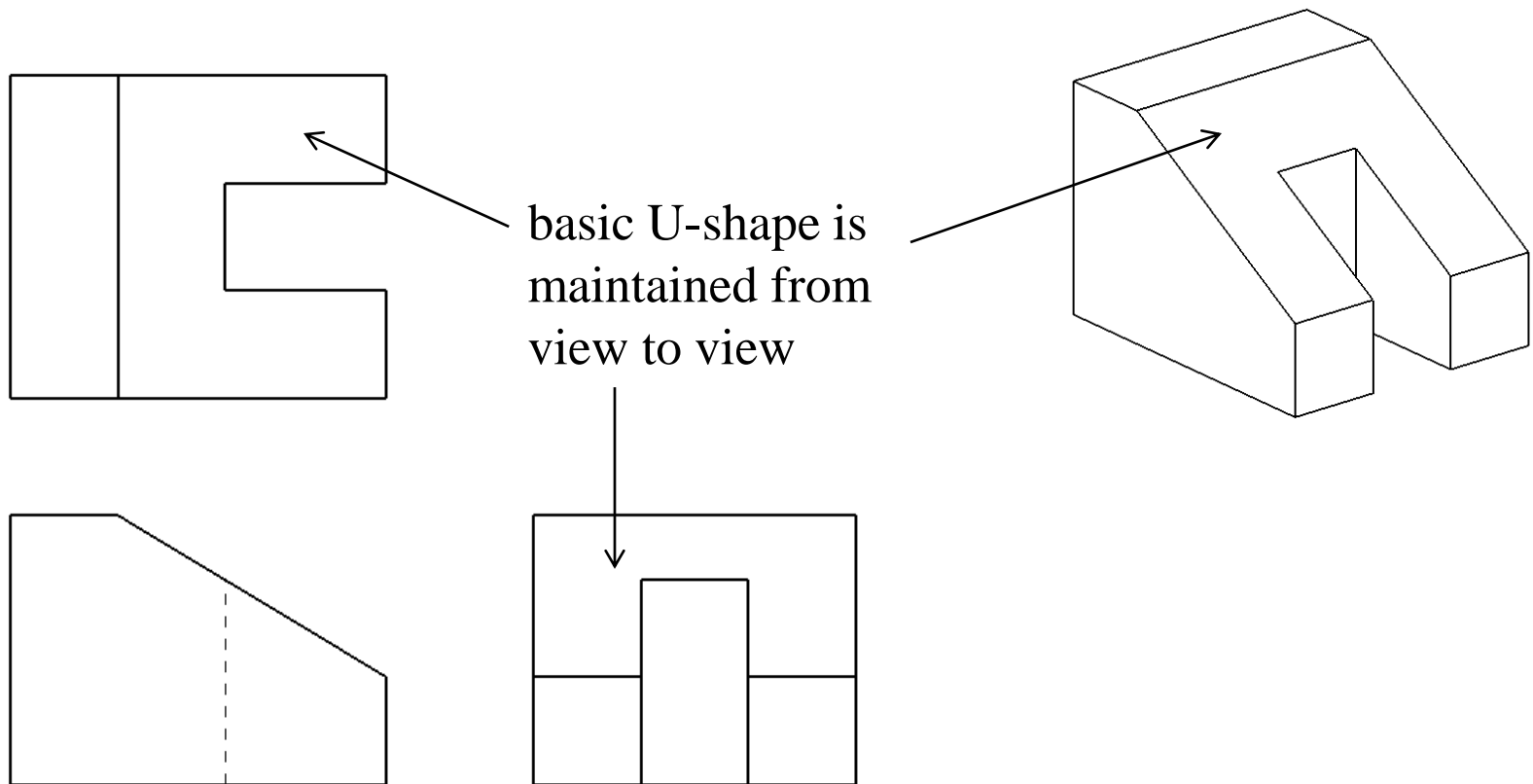
- Area views are *foreshortened*
- Basic shape is maintained in area views

# Normal versus Inclined Surfaces



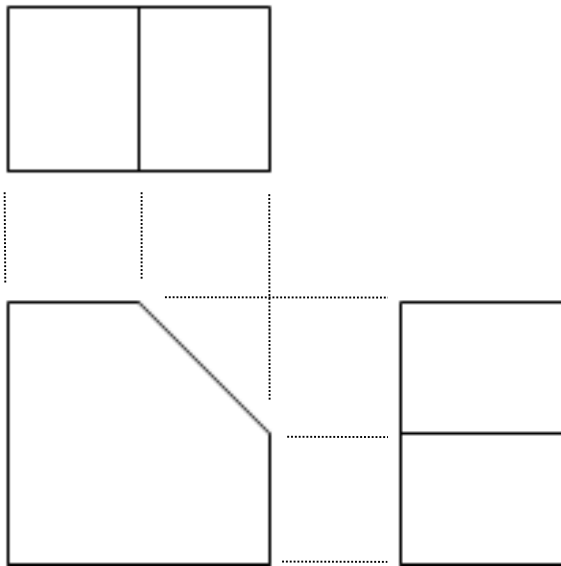
# Inclined Surfaces

- An inclined surface maintains its basic shape from view to view

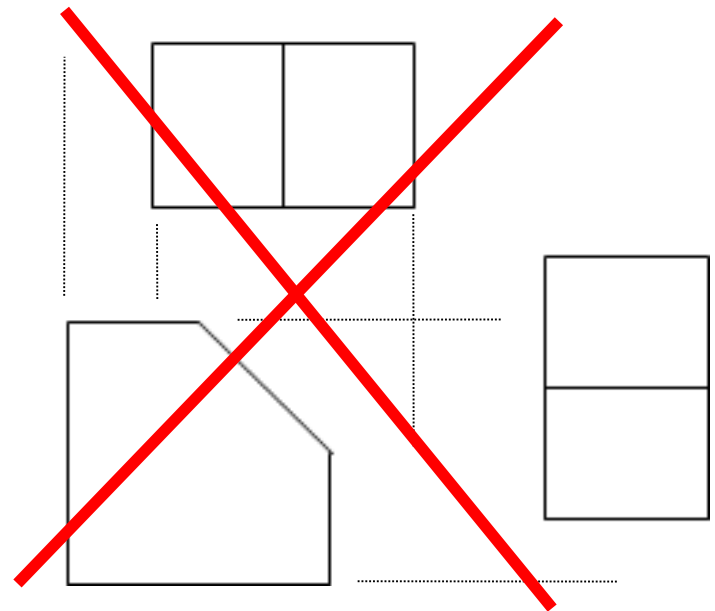


# Orthographic Projection

Recall: Align views with each other (features project from one view to the next)



Properly aligned  
orthographic views

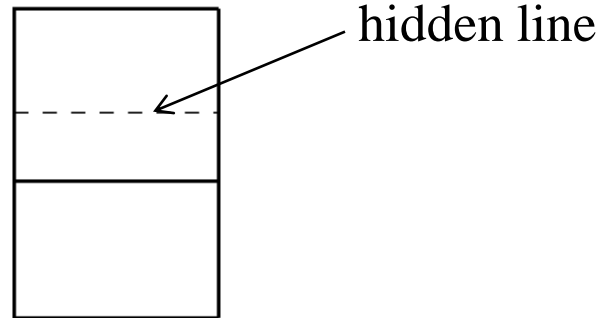
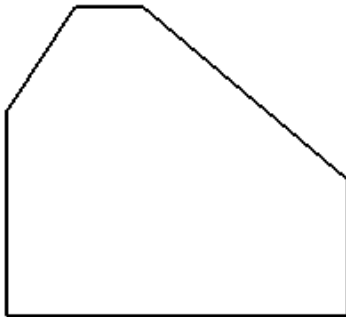
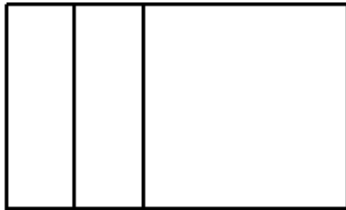


Improperly aligned  
orthographic views



# Orthographic Projection

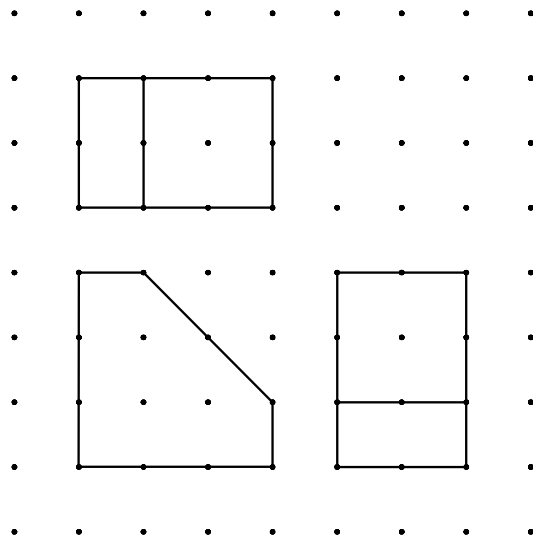
Recall: Use hidden (dashed) lines to show edges of an object that are hidden from a given viewpoint.



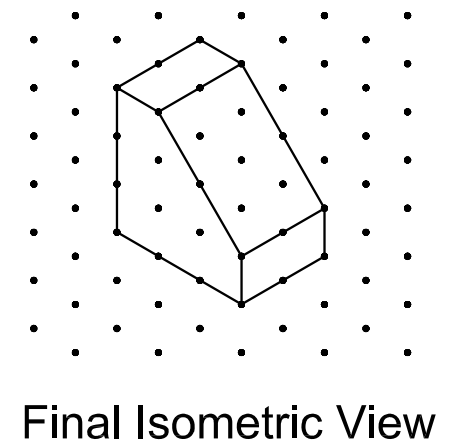
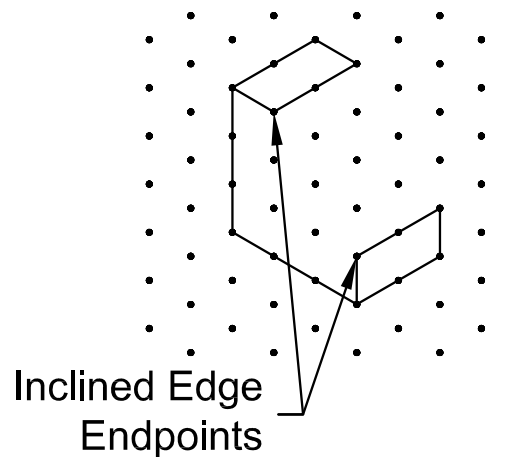
# Inclined Surfaces in Isometric

Constructing an isometric view of an inclined surface:

1. Locate the endpoints of each inclined edge
2. Draw a straight line between them

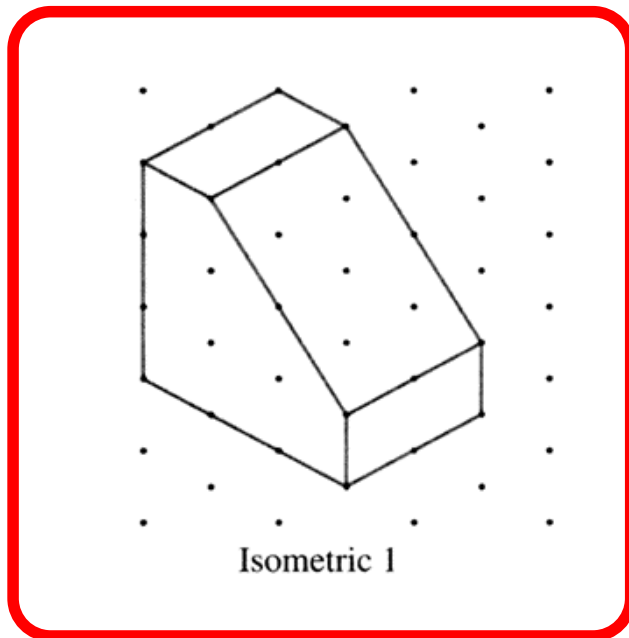


Orthographic Views

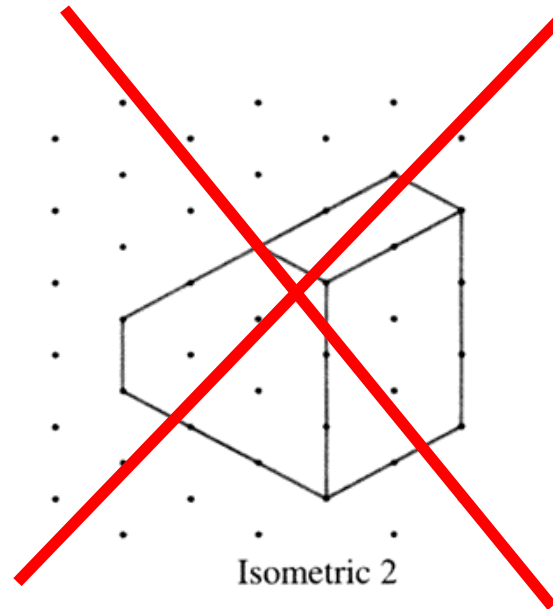


# Inclined Surfaces: Constructing Isometric Views

Select an orientation that makes the inclined surface appear as a visible area (whenever feasible)

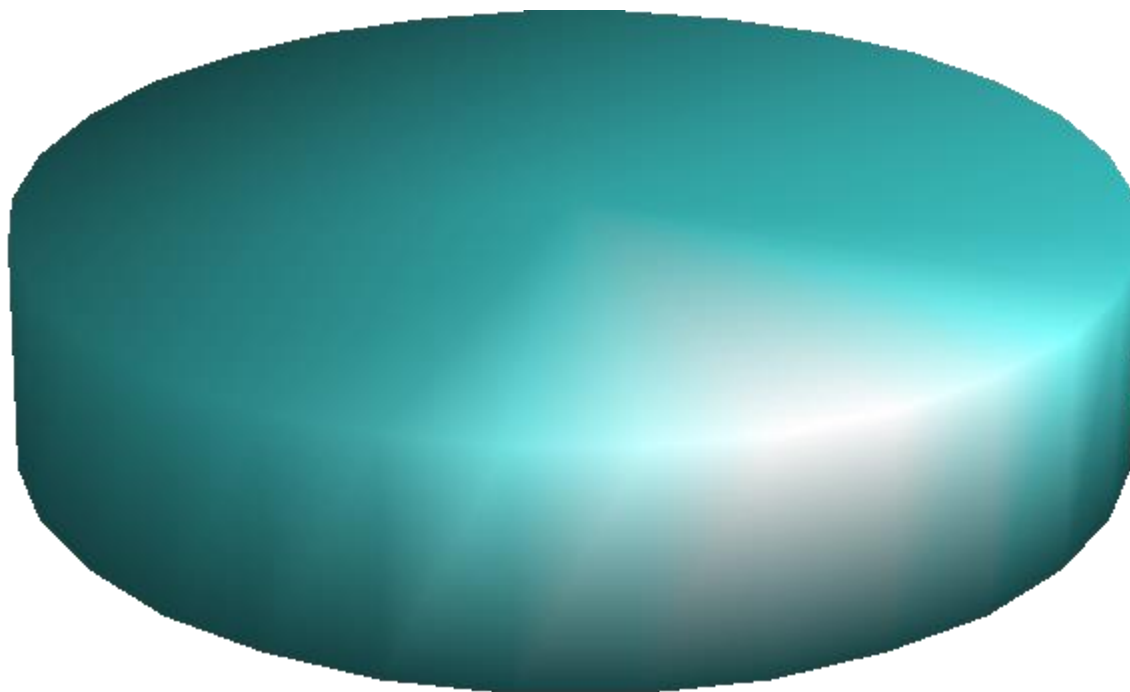


**Correct**



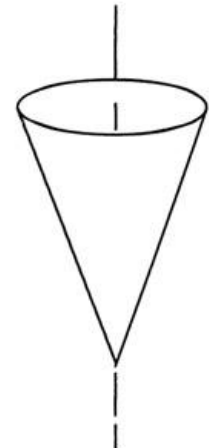
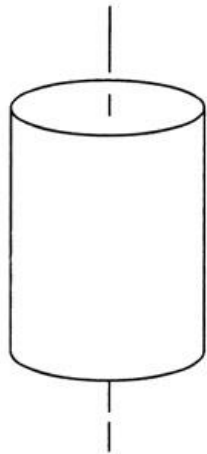
**Incorrect**

# Single Curved Surfaces



# Single Curved Surfaces

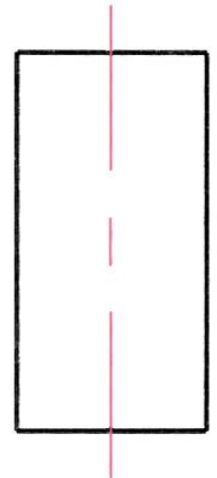
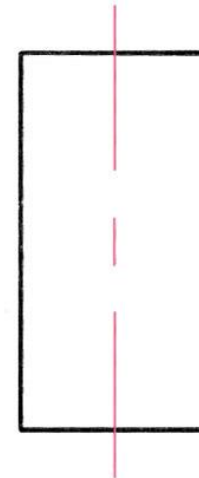
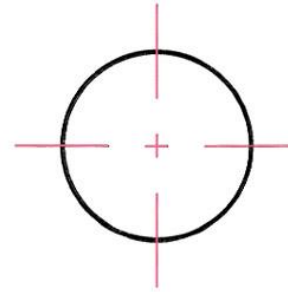
- Surfaces having a curvature about one axis.
- Generated by revolving a line about an axis.



# Single Curved Surfaces

## Consider a cylinder:

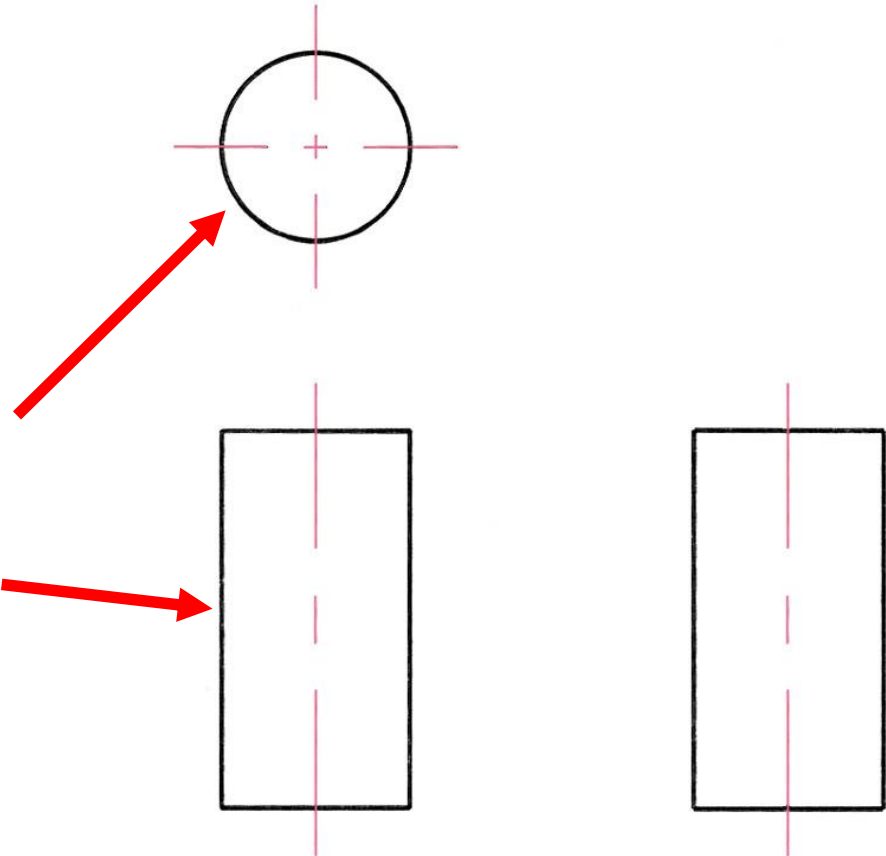
- Shown as a circle in one orthographic view
- Other orthographic views are rectangular
  - Rectangle width is equal to the cylinder diameter and represents the cylindrical boundary or visible extents of the surface



# Single Curved Surfaces

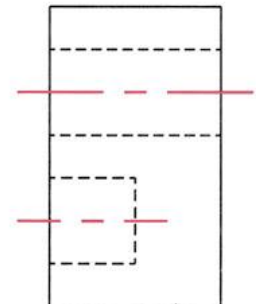
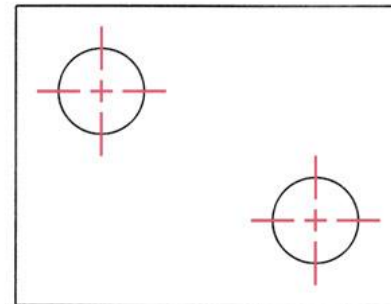
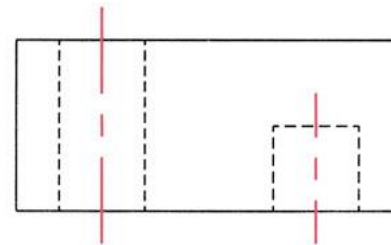
Because a curved surface is rectangular in two views, you **MUST** indicate the radial center with centerlines.

- Crossing centerlines are used in the circular view.
- A single centerline is shown in each “rectangular” view.



# Single Curved Surfaces-Holes

Holes follow the same rules as external curved surfaces, except the cylindrical boundaries are shown as hidden lines.

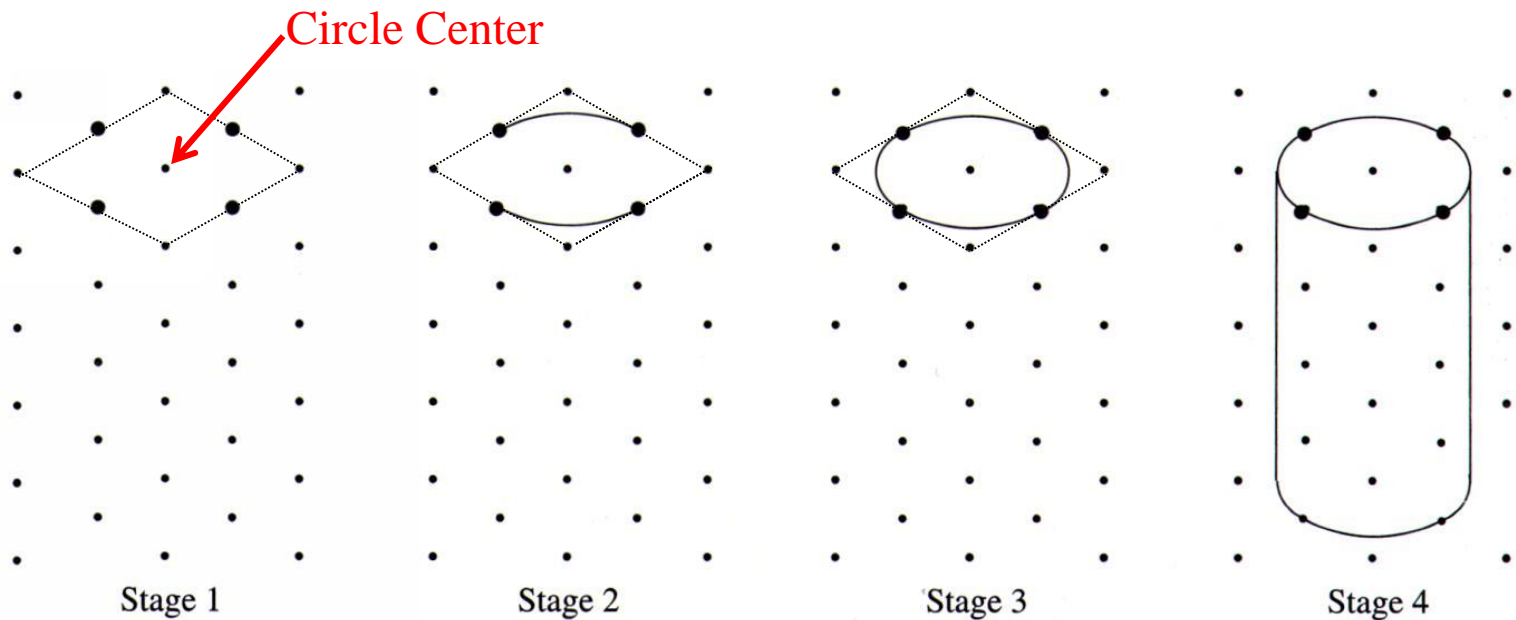




# Drawing Single Curved Surfaces in Isometric

A circle appears as an ellipse in an isometric view

1. Locate the center of the circle
2. Locate the radial points
3. Sketch the "bounding box" for the ellipse
4. Sketch the four arcs of the ellipse tangent to the box edges
5. Complete the rest of the object



Hi

VISUALIZING SUCCESS

# Course Software.....

- Work through the Inclined and Curved Surfaces Software Module

# Orthographic Projection of Inclined & Curved Surfaces Homework ...

- Complete the following pages in Module 5  
inc/crv-
  - 1 - 2
  - 4 - 5
  - 7 - 9
  - 11 - 13
  - 15 - 16

# Flat Patterns

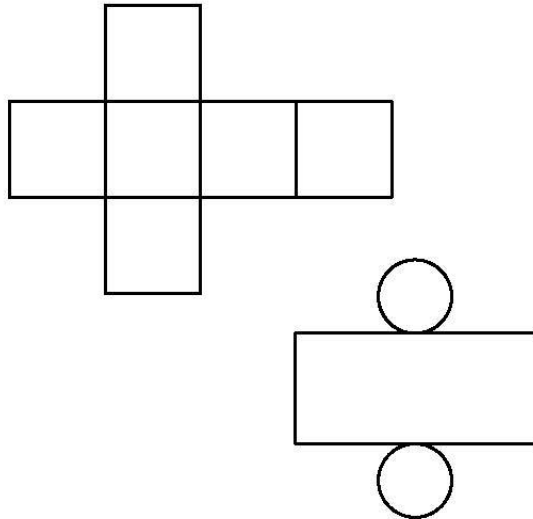
## Module 6

# Session Topics

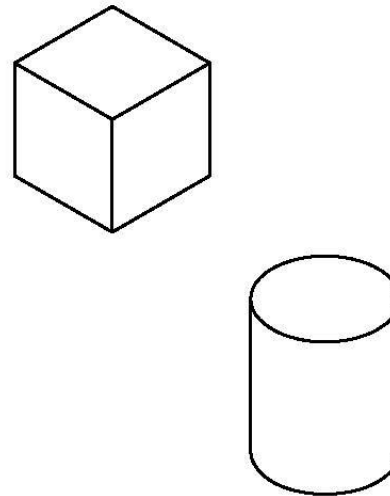
- Flat Patterns
- Fold Lines
- Multiple Patterns
- Open Surfaces
- Closed Surfaces
- Patterns with markings

# Flat Patterns

- Sometimes it is important to visualize how a flat pattern can be folded up to obtain a 3-D object



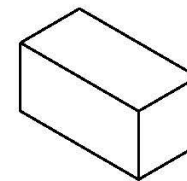
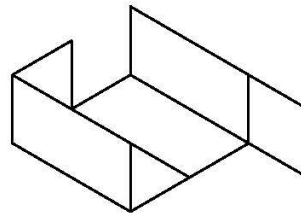
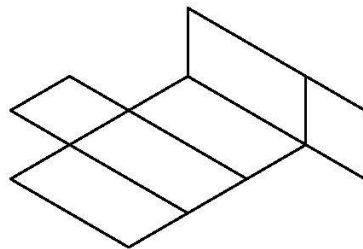
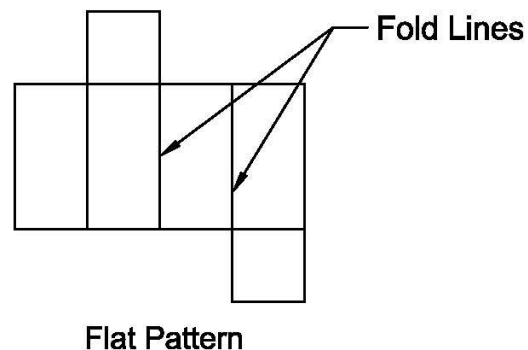
2-D Patterns



3-D Objects

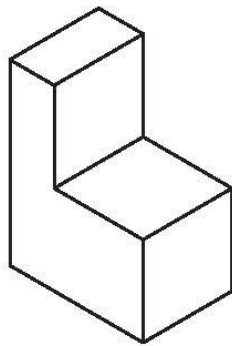
# Fold Lines

- The solid lines on a flat pattern are *fold lines*
  - When visualizing creating a 3-D object from a flat pattern, think about folding it at the fold lines

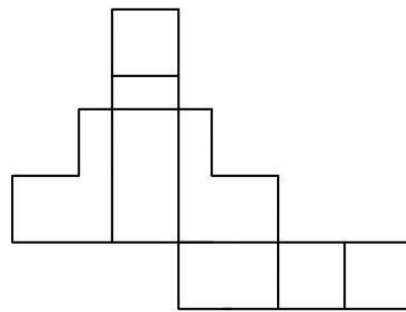


# Multiple Patterns

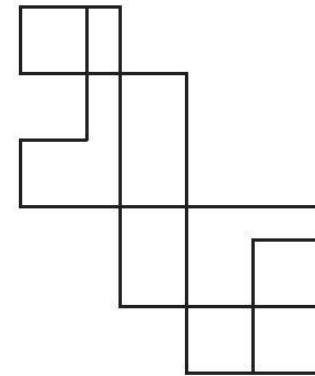
- Most objects have more than one pattern that could be folded to form it



3-D Object



Pattern 1



Pattern 2

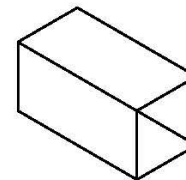
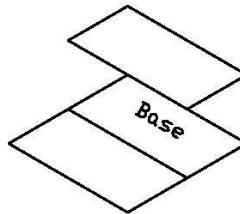
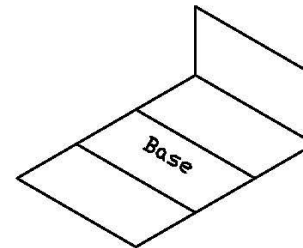
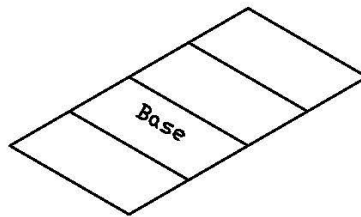


# Open Objects from Patterns

- No "ends" are included in the pattern, so it is folded up and an open object results
  - Tube from paper towels



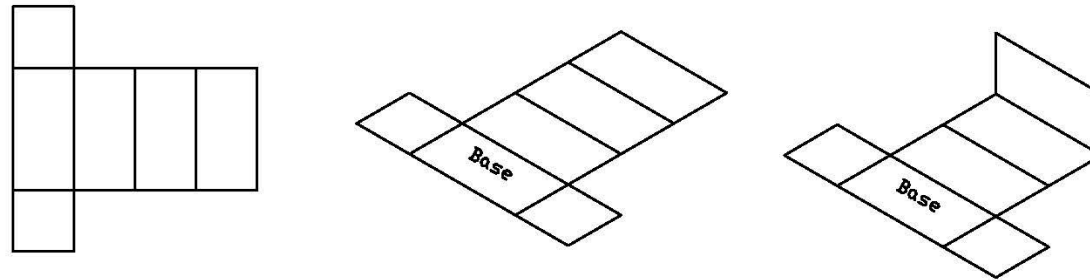
Pattern for Hollow Object



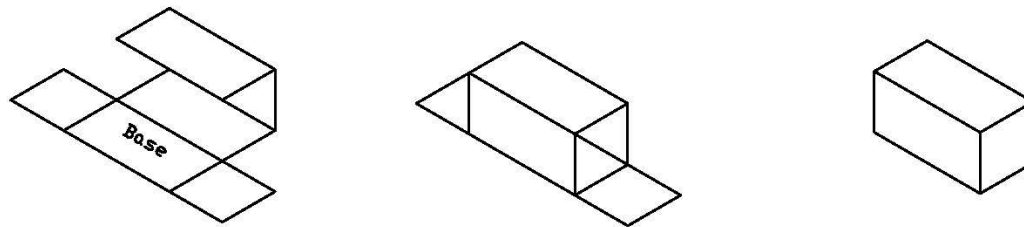
Pattern Folder to Form Hollow Object

# Closed Objects from Flat Patterns

- When the pattern includes "ends" a closed surface (object) results



Pattern for Object with Closed Ends

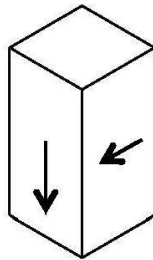
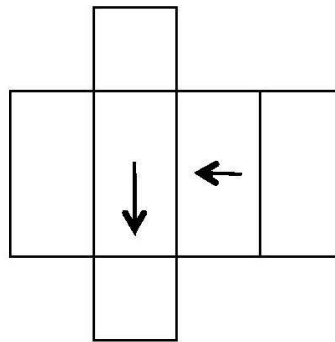


Pattern Folded to Form Closed Object

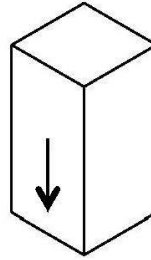
# Markings on Patterns

- When there are markings on a pattern that are on adjacent sides
  - Markings must end up adjacent to one another on the object
  - Markings must end up in the same orientation on the object

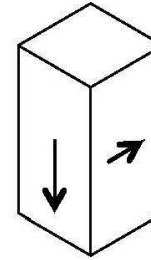
# Markings on Patterns (continued)



Correct 3-D Object



Markings not on  
Adjacent Surfaces



Markings on Adjacent  
Surfaces in Wrong  
Orientation

# Course Software...

- Work through the Flat Patterns software module

# Flat Patterns Homework ...

- Complete the following pages in Module 6
- fp-
  - 1 - 2
  - 5 - 6
  - 9 - 12
  - 17 - 22

# Rotation of Objects about a Single Axis

## Module 7

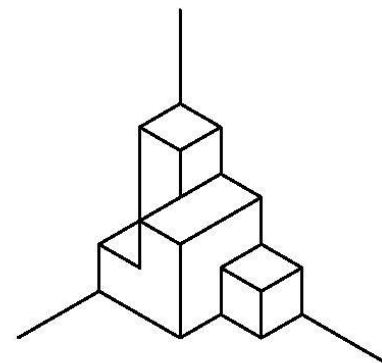
# Session Topics

- Object Rotations
- Right Hand Rule
- Rotation Notation
- Single Rotation
- Multiple Rotations
- Equivalent Rotations
- Computer Module

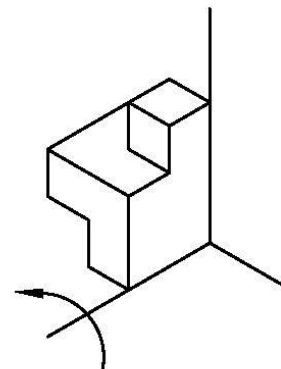
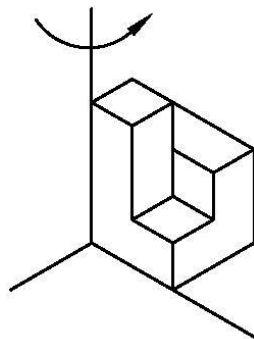
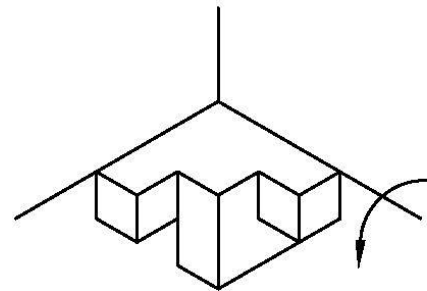


# Object Rotation

- A rotation is a turning of an object about a straight line known as the axis of rotation.

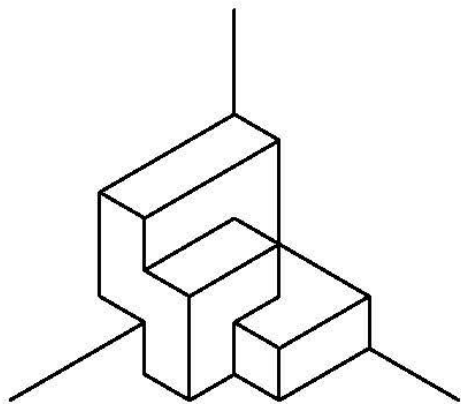


Original Object Position

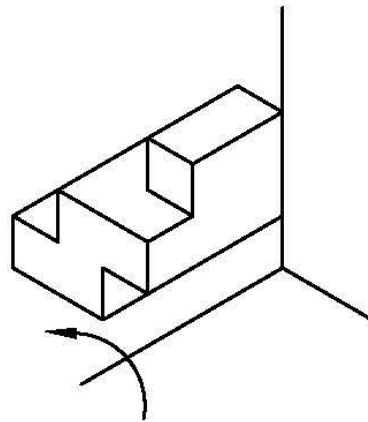


# Direction of Rotation

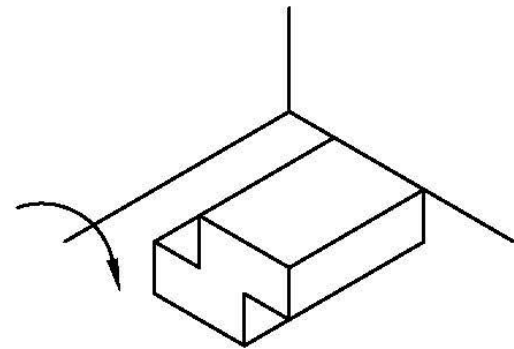
- A rotation about an axis can be either positive (counterclockwise) or negative (clockwise)



Original Object Position



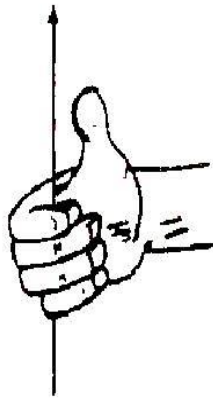
Positive Rotation



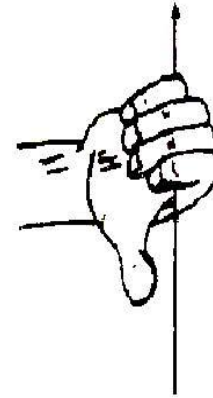
Negative Rotation

# Right Hand Rule

- If you place the thumb of your *right hand* down the axis of rotation, your fingers will curl in the direction of the rotation





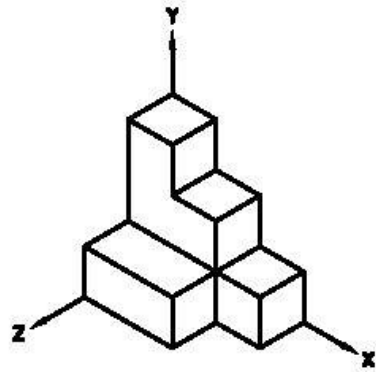
Positive Rotation



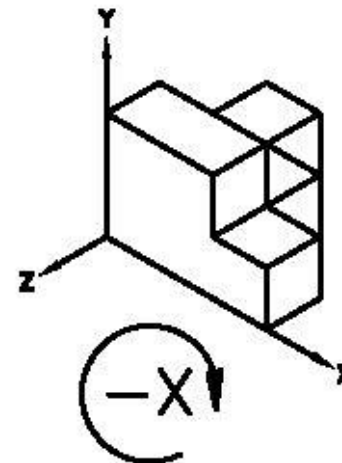
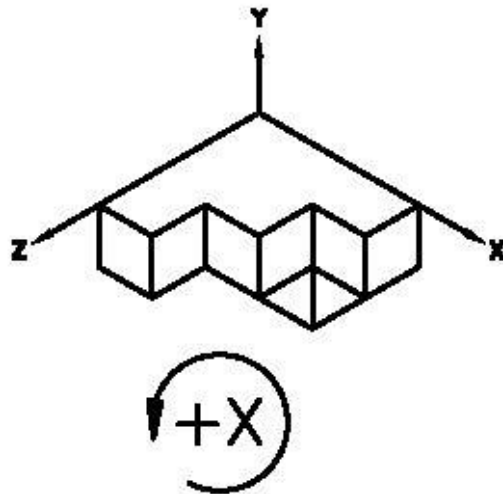
Negative Rotation

# Arrow Coding

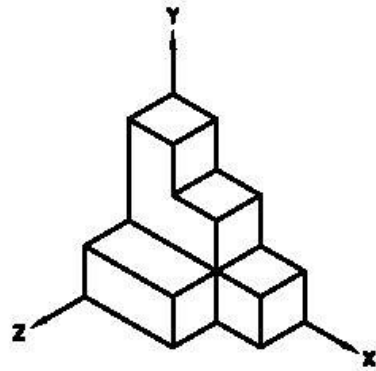
- Object Rotations can be designated by arrow coding
  - A curved counterclockwise arrow is a positive rotation  
A diagram showing a circle with a curved arrow pointing counterclockwise. Inside the circle is a plus sign (+) and an 'X' representing the axis of rotation.
  - A curved clockwise arrow is a negative rotation  
A diagram showing a circle with a curved arrow pointing clockwise. Inside the circle is a minus sign (-) and an 'X' representing the axis of rotation.
  - The axis for the rotation is included within the notation
  - The increment for the rotation is always 90°



Original Object Position

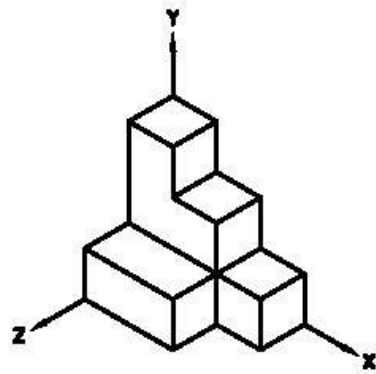


For each of the following slides,  
try to visualize the rotation

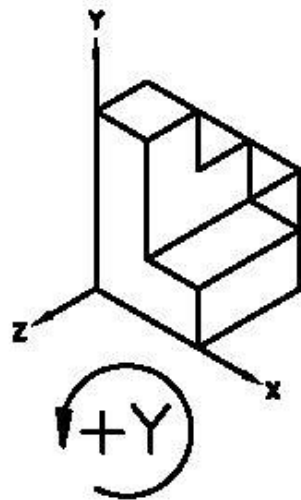


**Original Object Position**

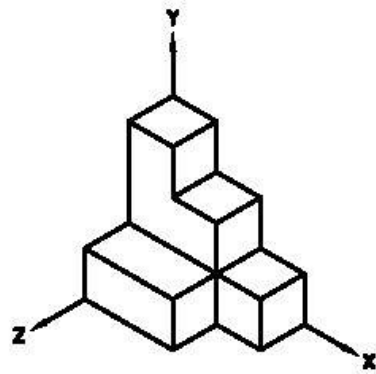




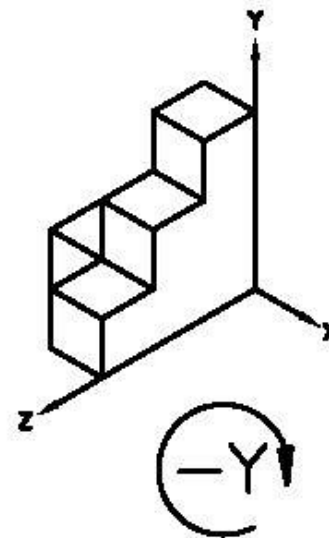
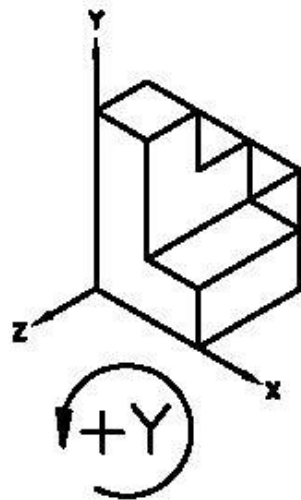
Original Object Position

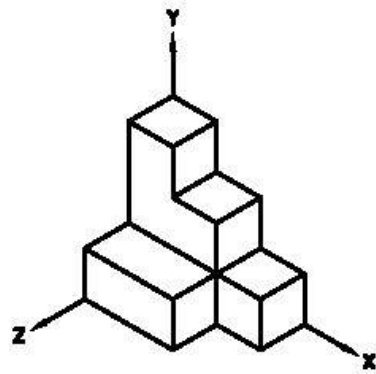






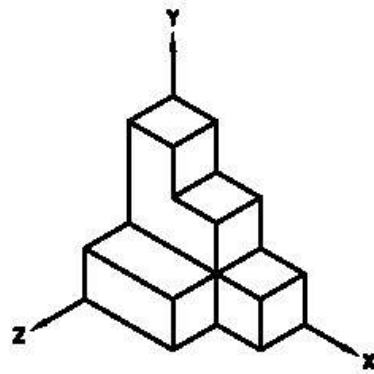
Original Object Position



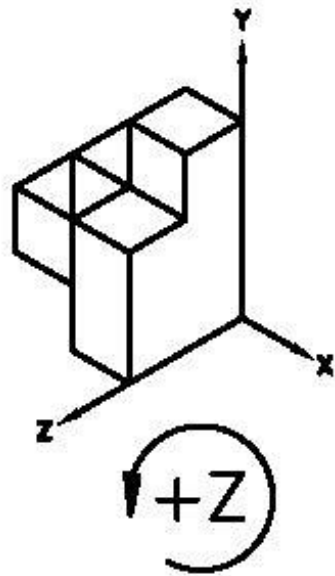


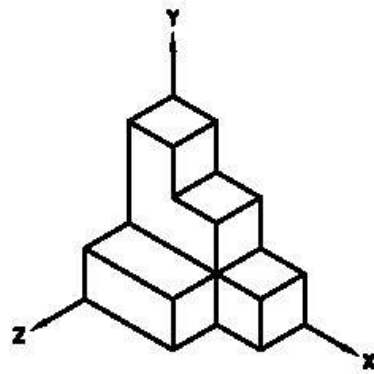
Original Object Position



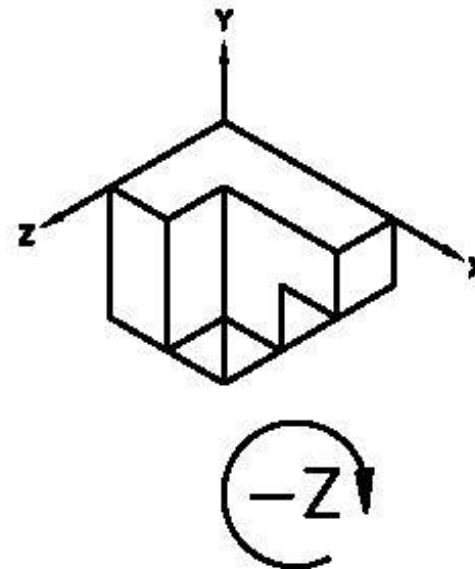
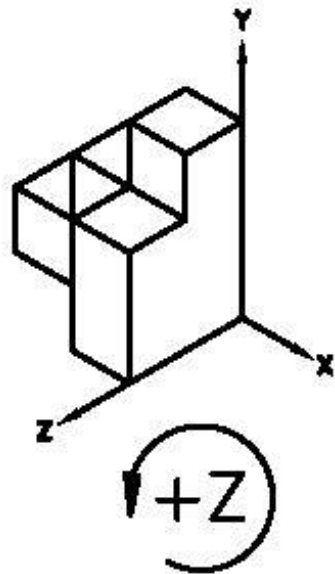


Original Object Position



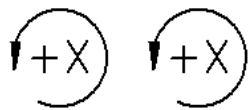


Original Object Position



# Multiple Rotations

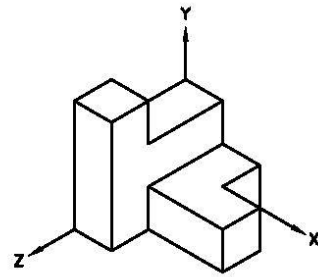
- For multiple rotations about the same axis, use an arrow for *each* rotation of  $90^\circ$ .



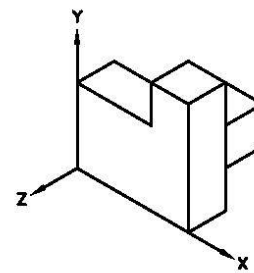
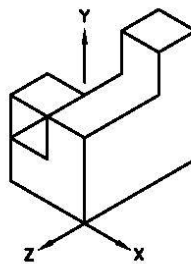
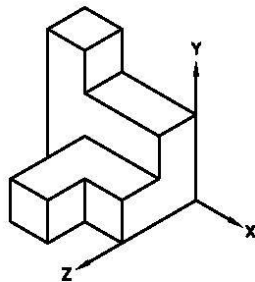
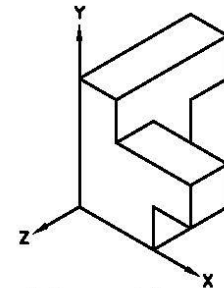
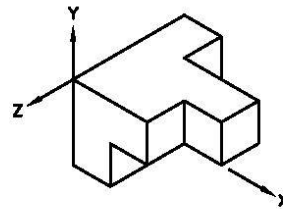
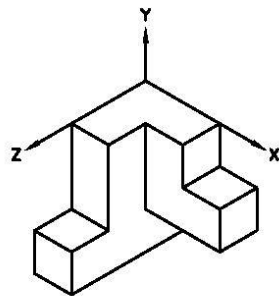
Positive  $180^\circ$  rotation about the x-axis



Negative  $270^\circ$  rotation about the y-axis

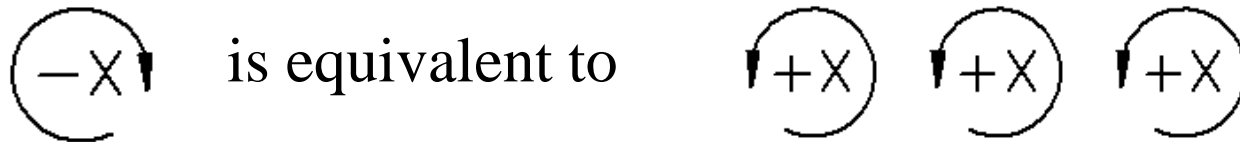


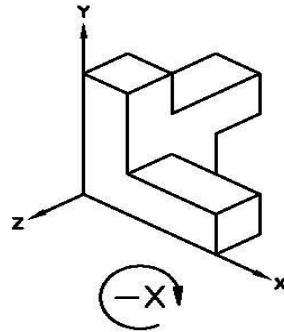
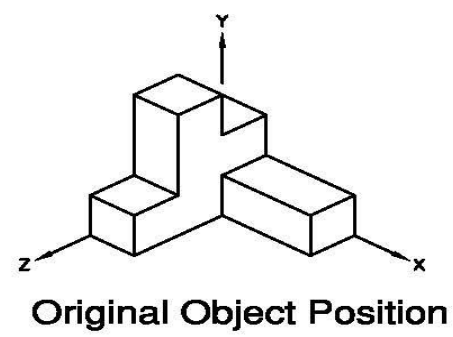
Original Object Position



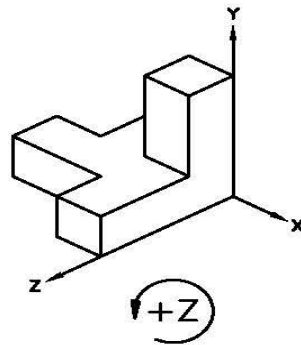
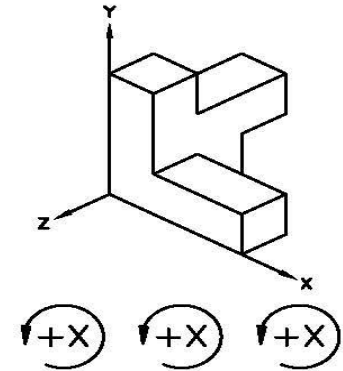
# Equivalent Rotations

- Sometimes one set of rotations can be replaced by a simpler set.

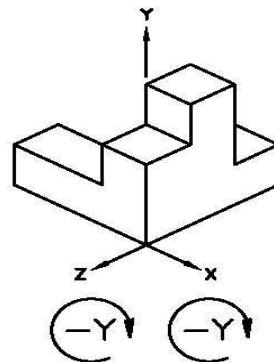
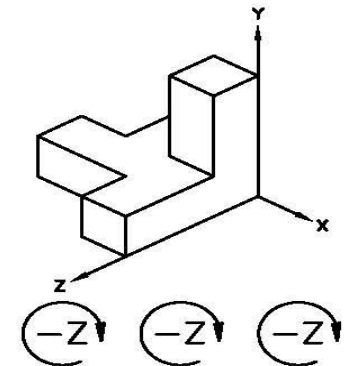




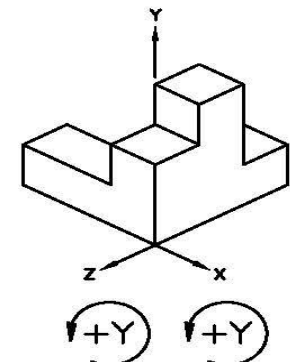
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=

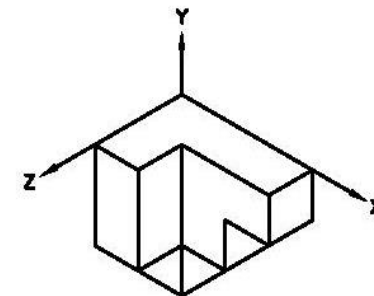
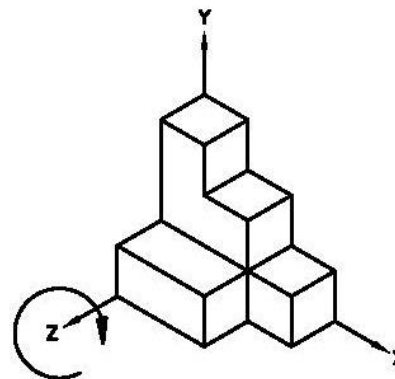
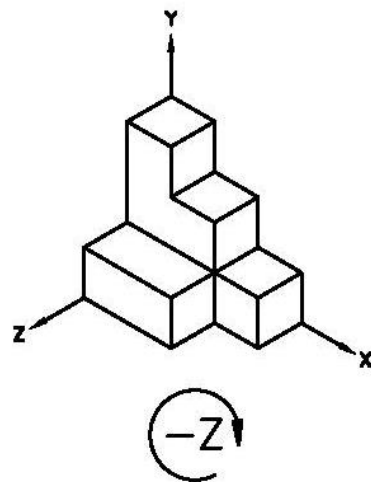
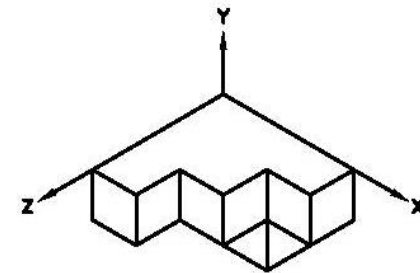
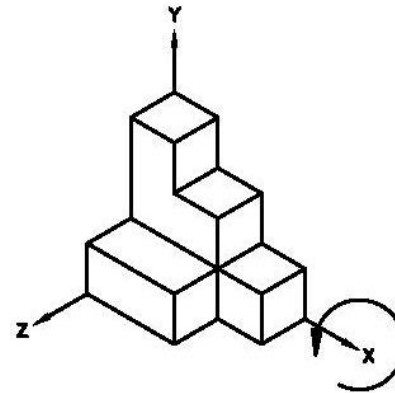
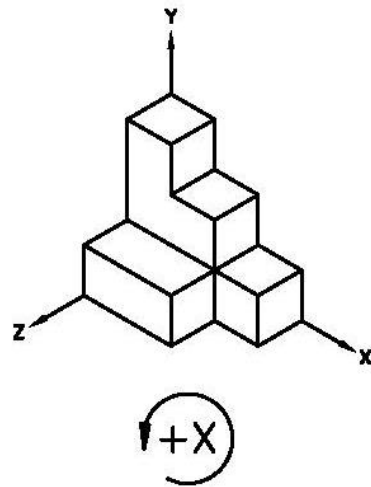


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To visualize a rotation, think about moving the notation to the *positive* end of the axis—the arrow will show you the direction of the rotation



# Course Software...

- Work through the Rotation of Objects About a Single Axis computer module

# Rotation of Objects about a Single Axis Homework ...

- Complete the following pages in Module 7
- rot1-
  - 1 - 3
  - 5 - 6
  - 9 - 10
  - 13 - 14
  - 16 - 18

# Rotation of Objects about Two or More Axes

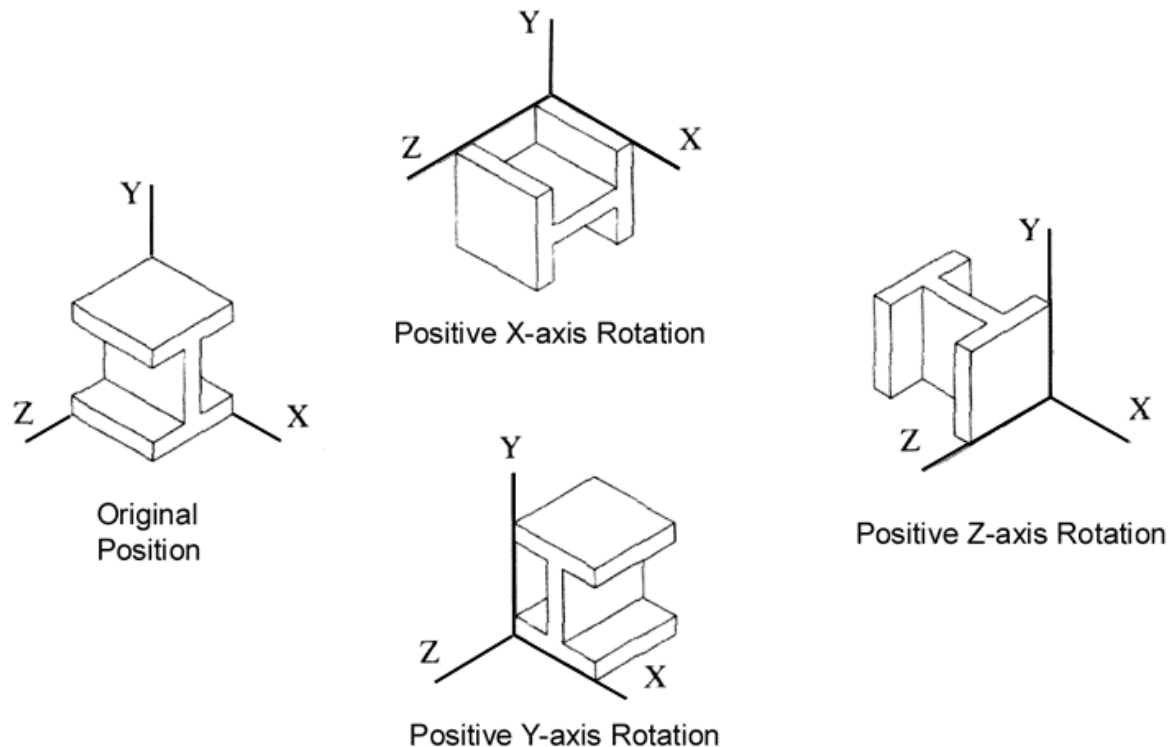
Module 8

# Session Topics

- Rotations about Two Axes
- Order of Rotations
- Equivalent Rotations
- Computer Module

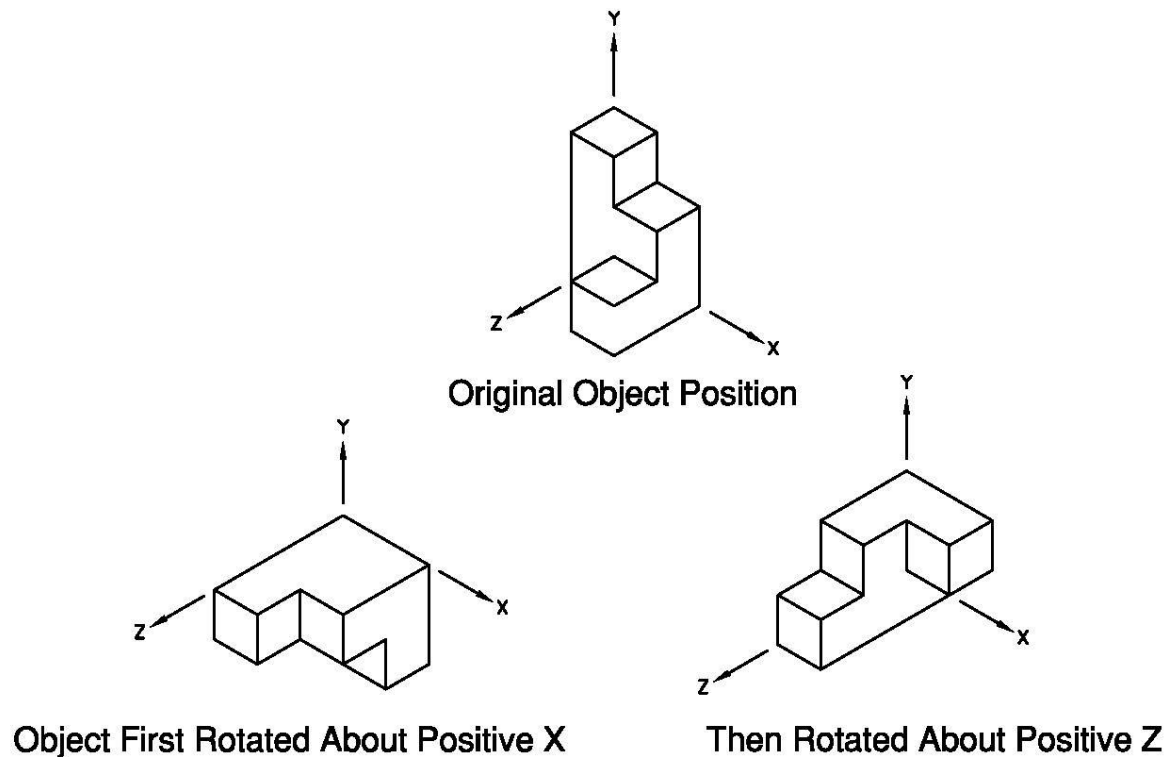
# Review: Rotation of Objects

- Rotation: turning an object about a straight line (axis of rotation)



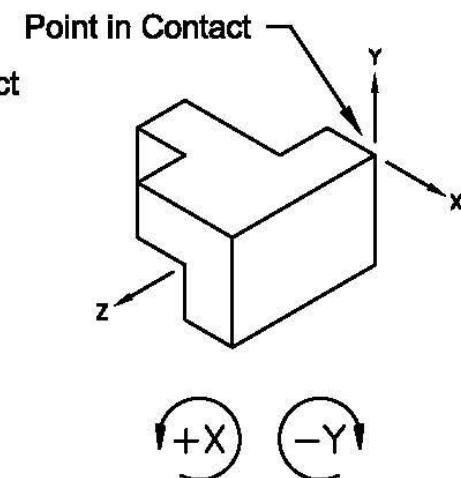
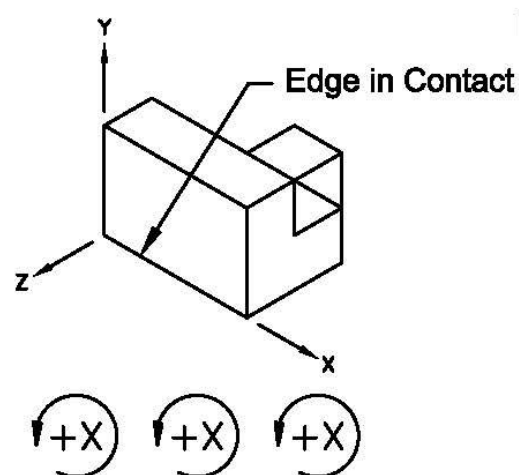
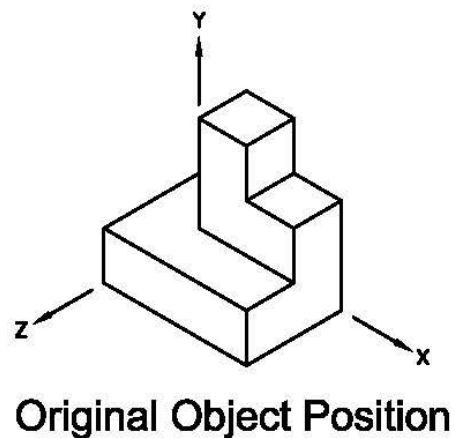
# Rotation About Two Axes

- Objects can be rotated about two or more axes the same way they were rotated about a single axis



# Rotation Origin

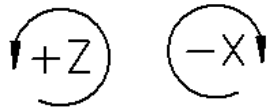
- When objects are rotated about two or more axes, only a single point remains in its original position





# Arrow Coding in Multiple-Axes Rotations

- Arrows are placed in the order in which the rotations are performed



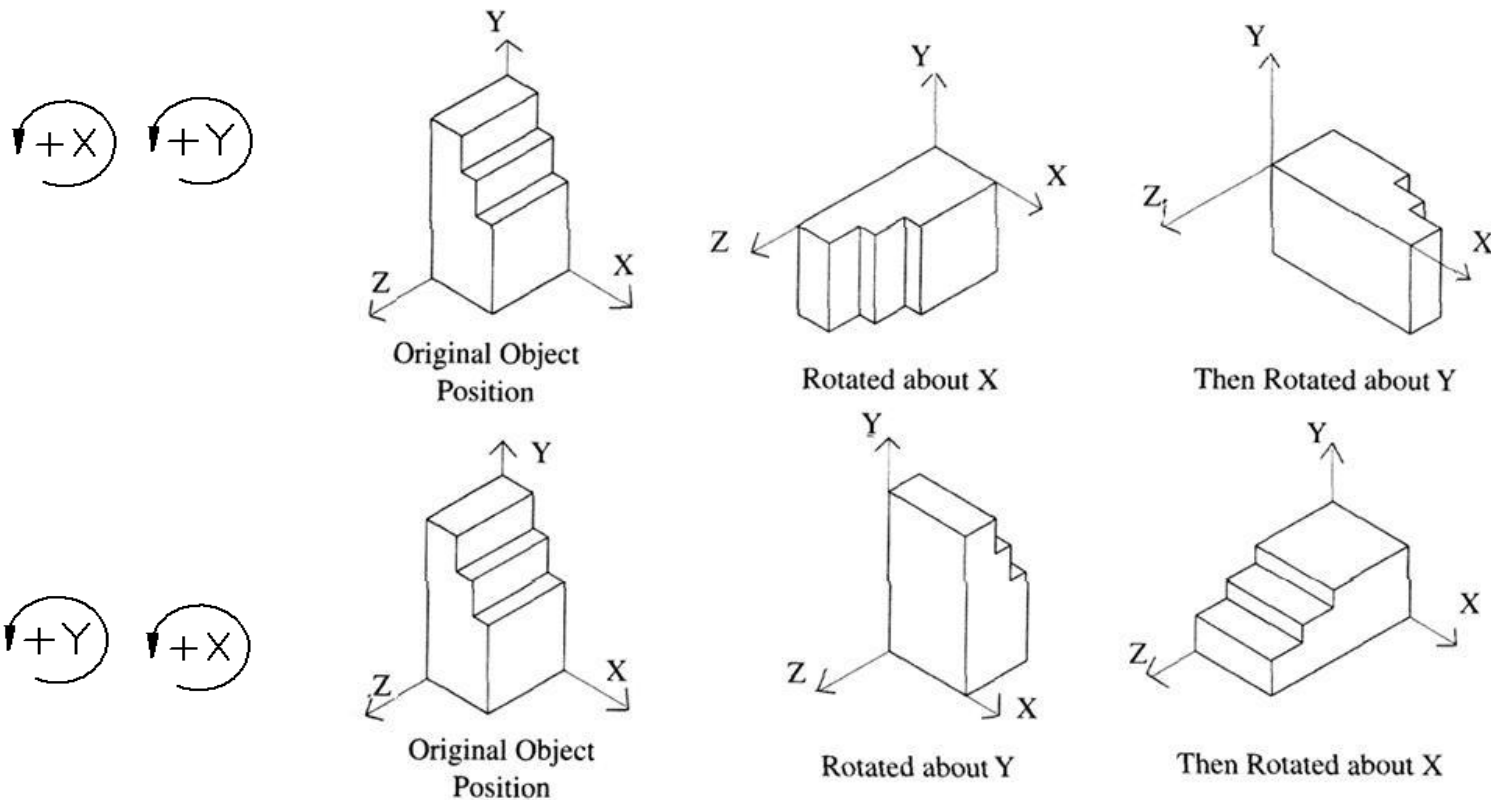
Positive 90° rotation about the z-axis followed by a negative 90° rotation about the x-axis.



Positive 90° rotation about the y-axis followed by a positive 90° rotation about the x-axis followed by a negative 90° rotation about the x-axis.

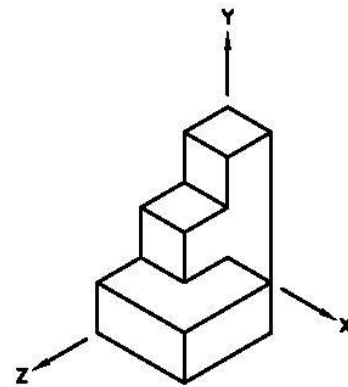
# Order of Rotations

- Final orientation of the object depends on the order in which the rotations were performed.

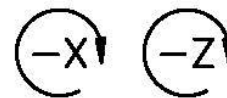
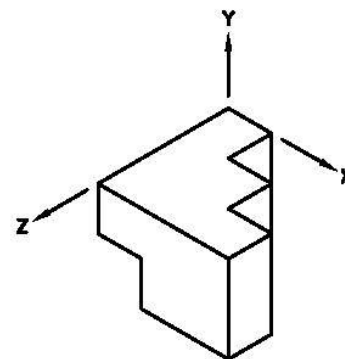
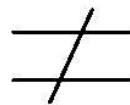
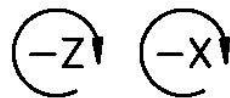
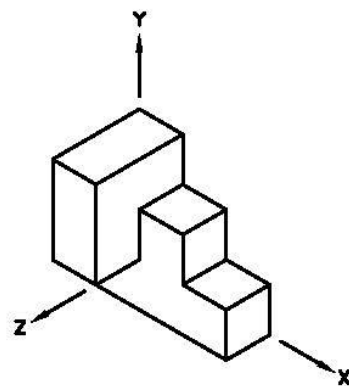


# Order of Rotations

- Object rotations about two or more axes are not commutative!

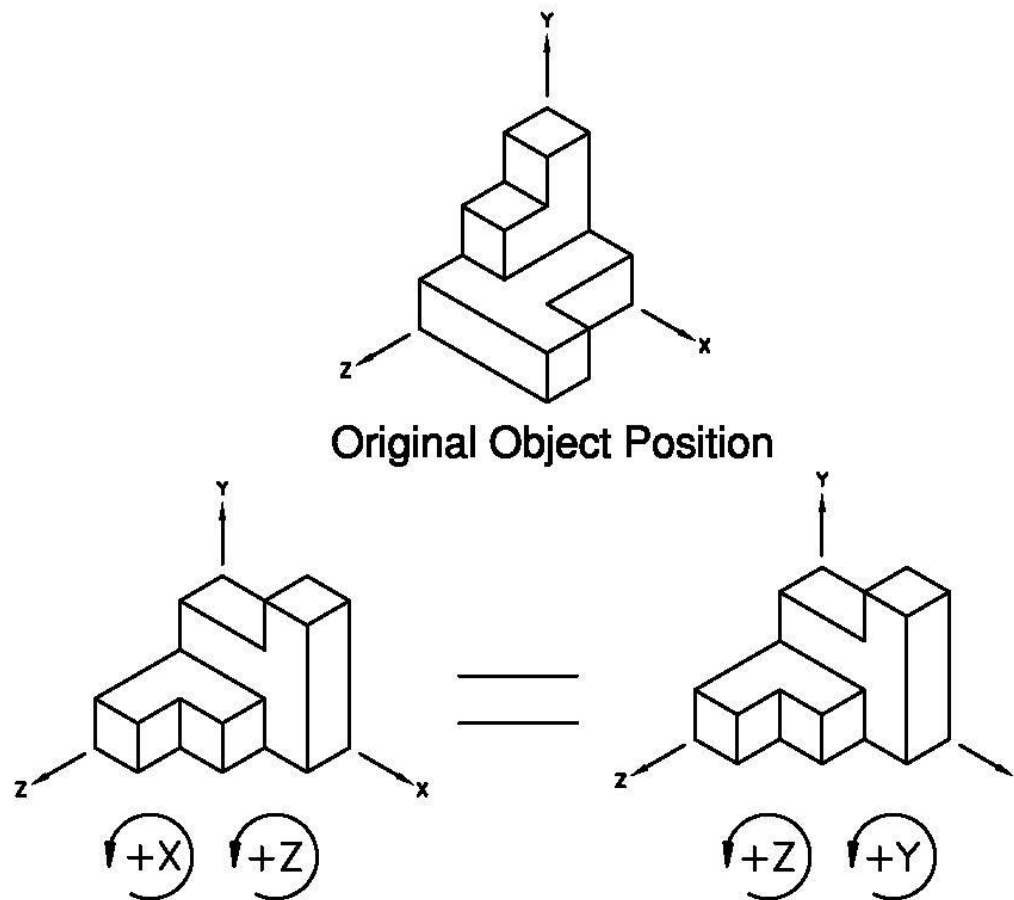


Original Object Position



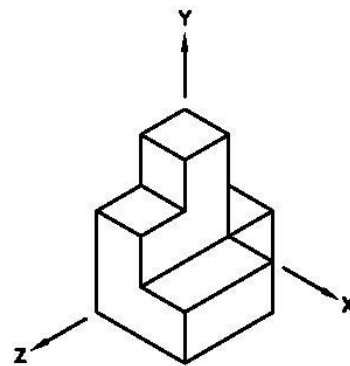
# Equivalent Rotations

- Two sets of rotations can result in the same final orientation of the object

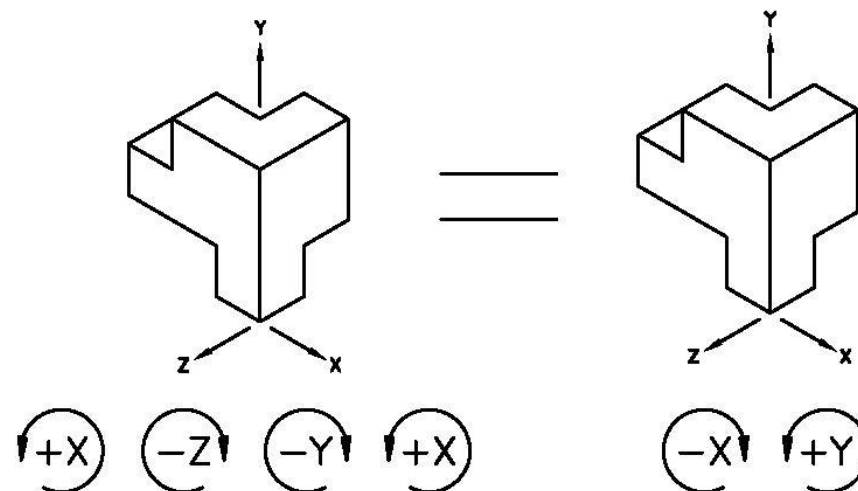


# Equivalent Rotations

- Sometimes one set of rotations can be replaced by a simpler set.



Original Object Position



# Course Software.....

- Work through the Rotation of Objects about Two or More Axes software module

# Rotation of Objects about Two or More Axes

## Homework ...

- Complete the following pages in Module 8
- rot2-
  - 1 - 2
  - 5 – 8
  - 11 – 12
  - 15 - 16

# Object Reflections and Symmetry

## Module 9

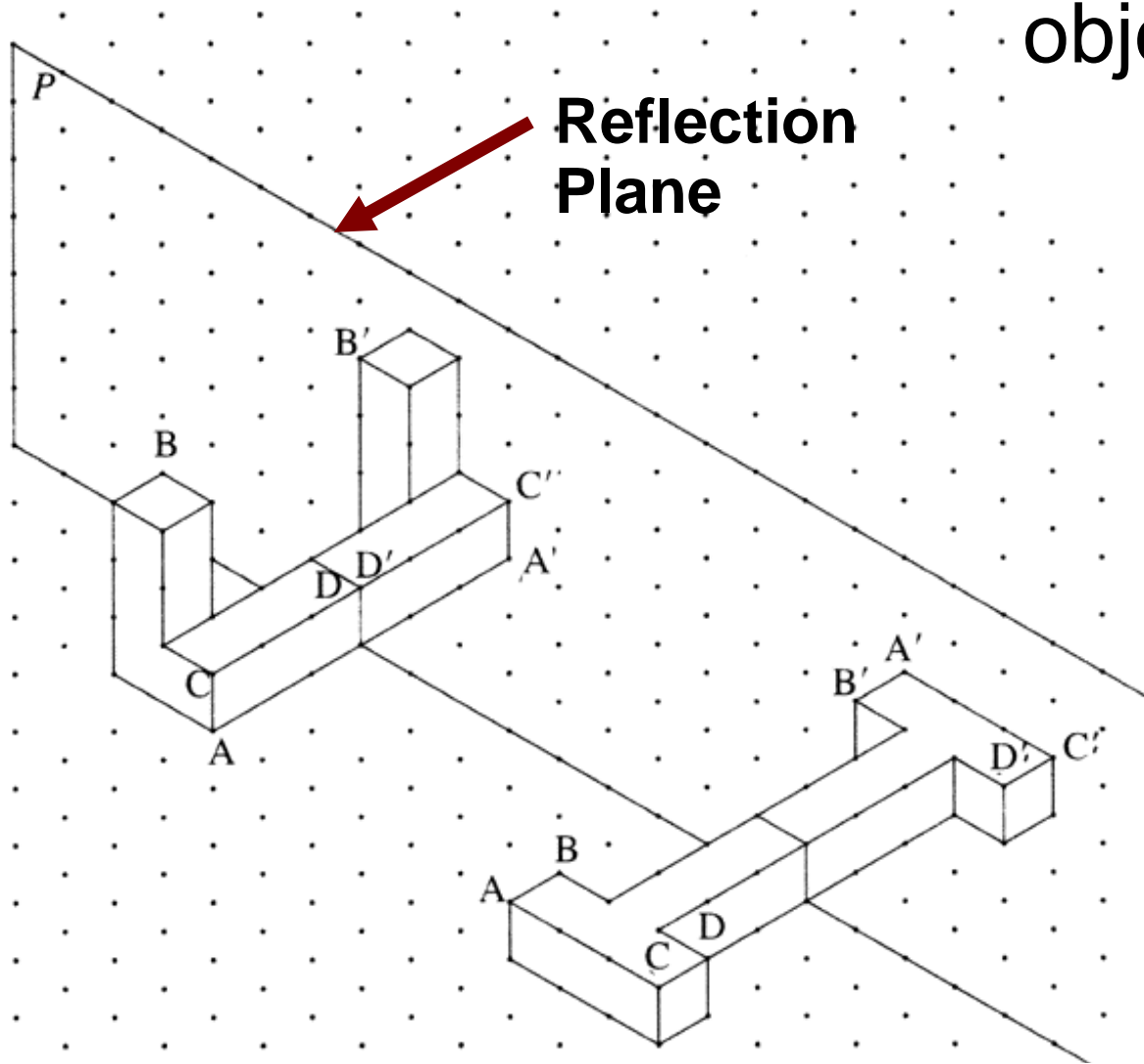


# Session Topics

- Reflection of an object
- Planes of symmetry
- Reflections through rotations

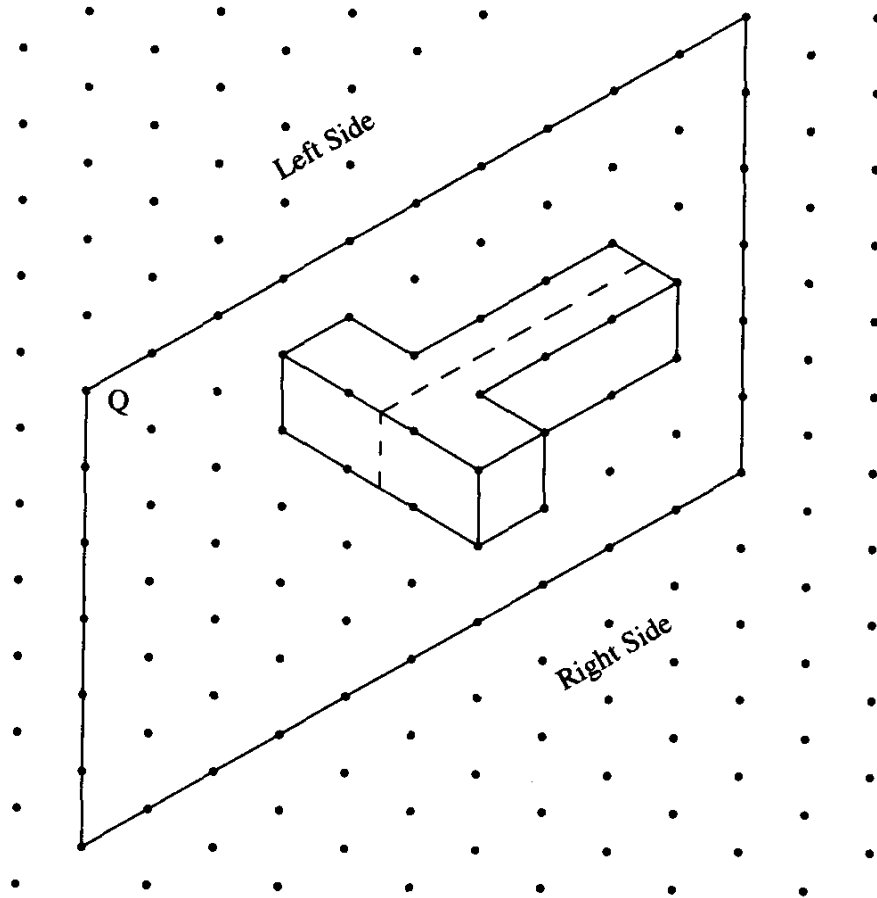
# Reflection

A reflection across a plane displays the object's mirror image.



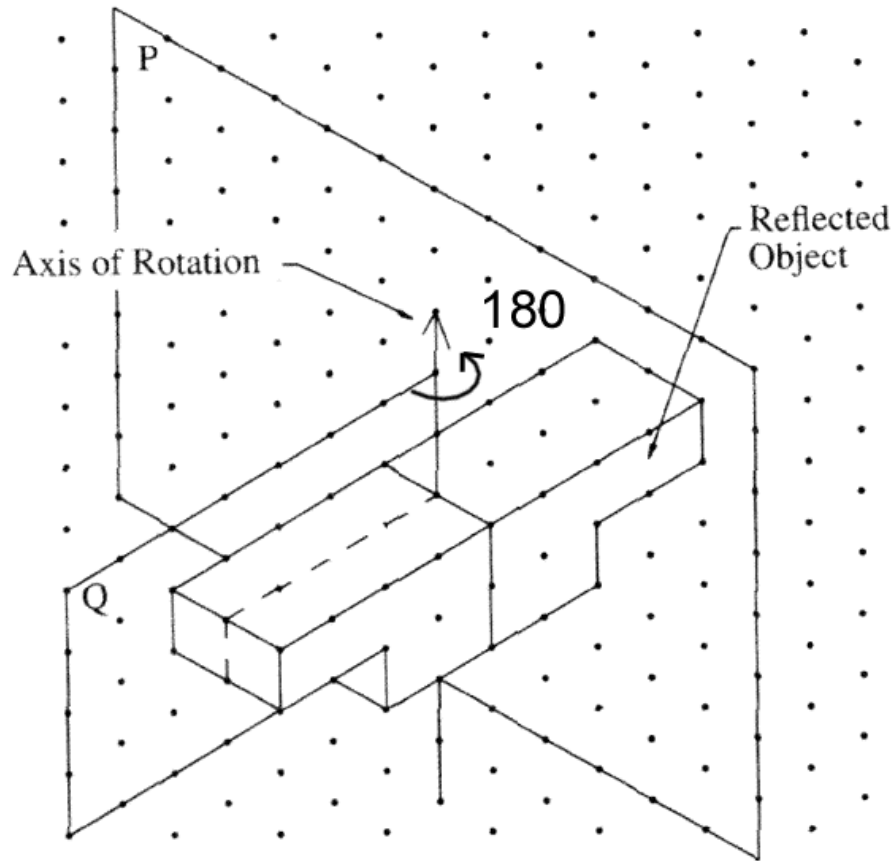
Each point,  $A$ , is associated with an image point,  $A'$ , such that the plane,  $P$ , is a perpendicular bisector of the line segment  $AA'$ .

# Plane of Symmetry



***Plane of symmetry*** occurs if the parts of the object on both sides of the plane are mirror images of each other.

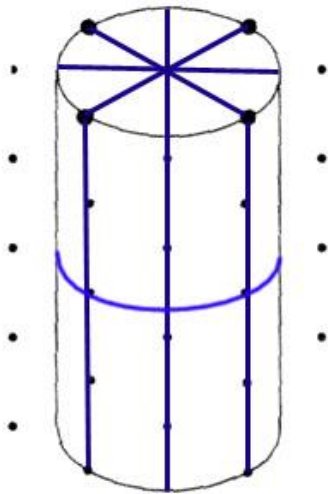
# Reflection through Rotation



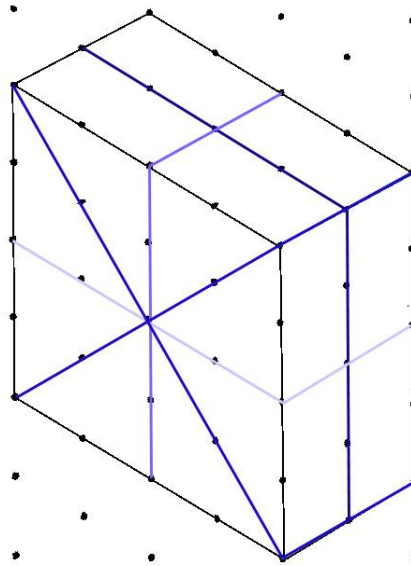
For a symmetric object, the mirror image can occur by rotating one side of the object  $180^\circ$  about an axis of rotation that's in the plane of symmetry.

# Multiple Planes of Symmetry

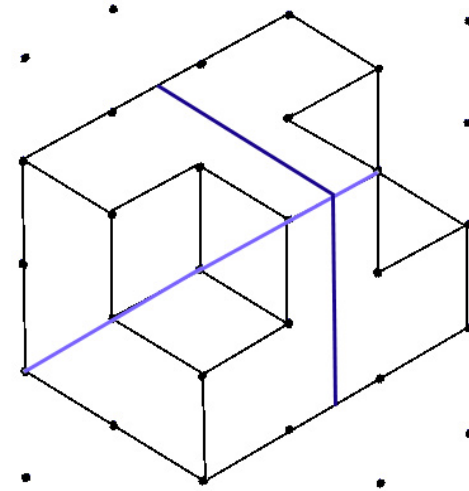
Many objects have multiple planes of symmetry



Infinite



5



2

# Course Software.....

- Work through the Reflections and Symmetry software module

# Object Reflections and Symmetry Homework ...

- Complete the following pages in Module 9
- reflx/sym-
  - 2 - 5
  - 8 - 9
  - 12 - 15

# Cutting Planes and Cross Sections

## Module 10

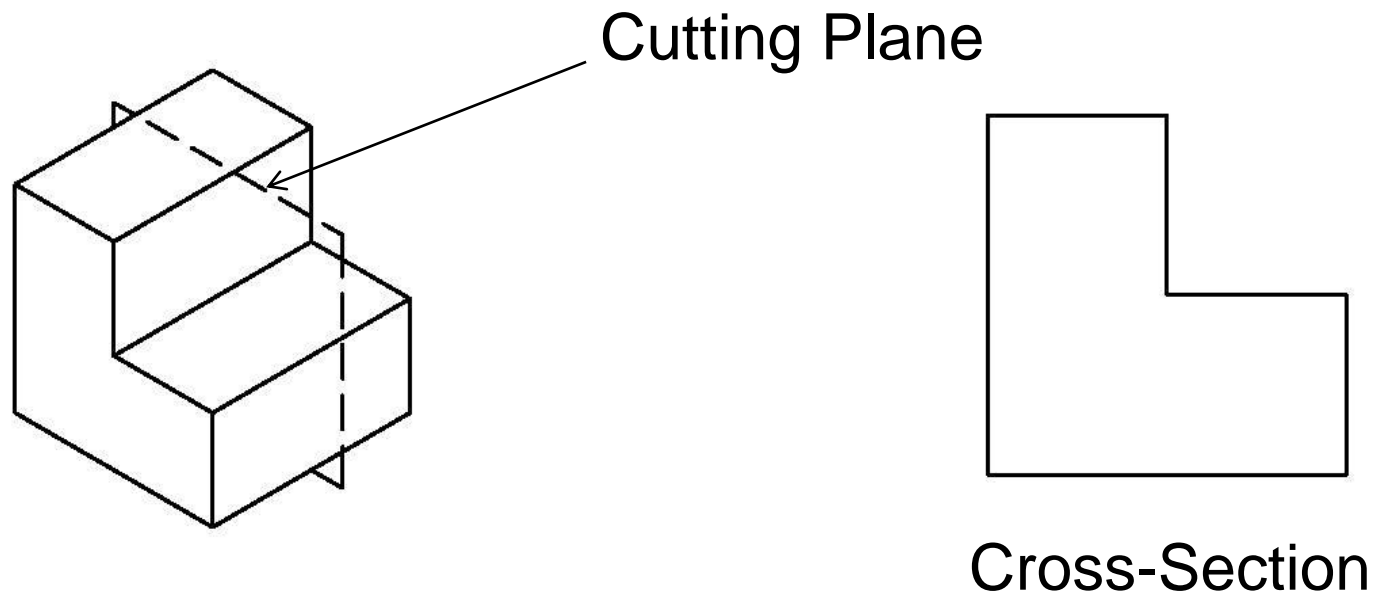


# Session topics

- Cutting planes
- Cross Sections
- Multiple Cross Sections

# Cutting Planes

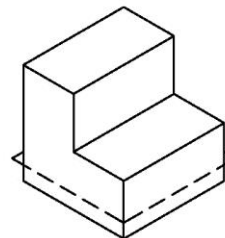
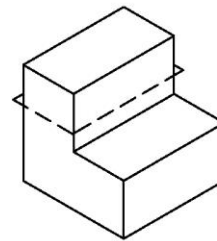
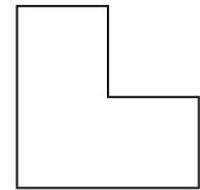
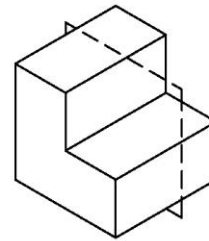
- A cutting plane is an imaginary plane that slices through an object



# Cutting Planes and Cross Sections

A cross section is the intersection of a cutting plane with a solid object.

- The result is a 2-D shape defined by the boundaries of the original object

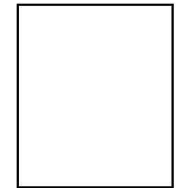
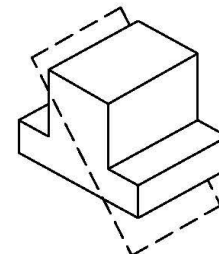
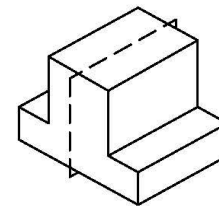
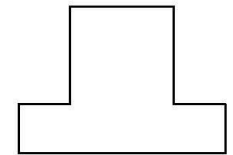
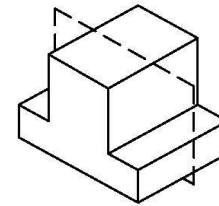
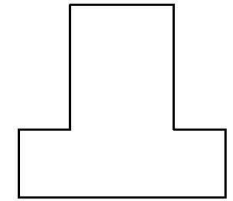
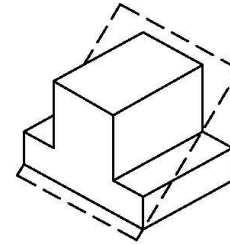


Object and Cutting Plane

Resulting Cross Section

# Cross Sections

The shape of the resulting cross section depends on the orientation of the cutting plane with respect to the object

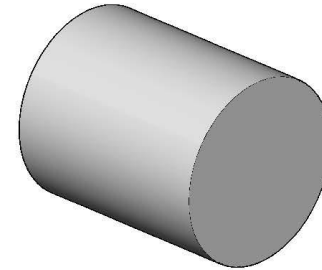


Object and Cutting Plane

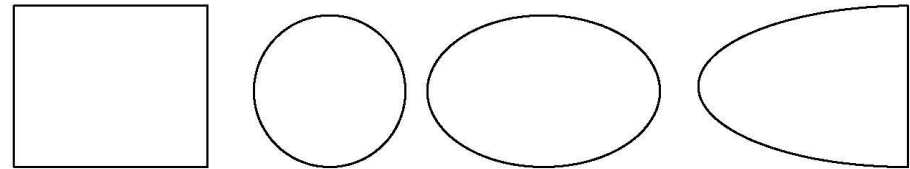
Resulting Cross Section

# Multiple Cross Sections

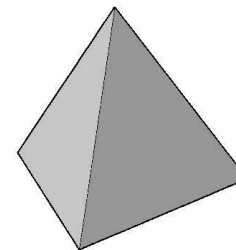
Objects can  
produce  
several cross  
sections



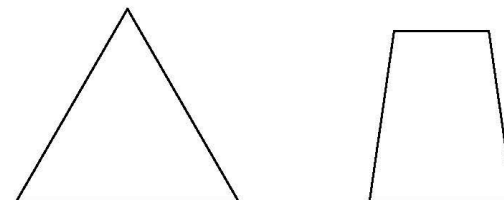
Object



Possible Cross Sections



Object

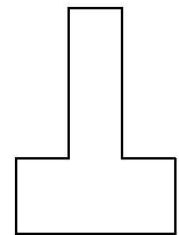
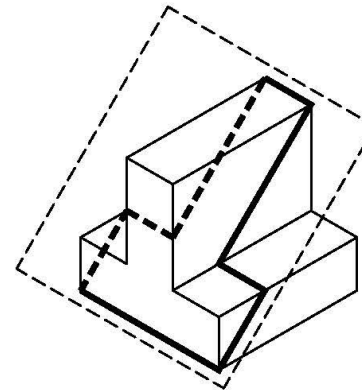
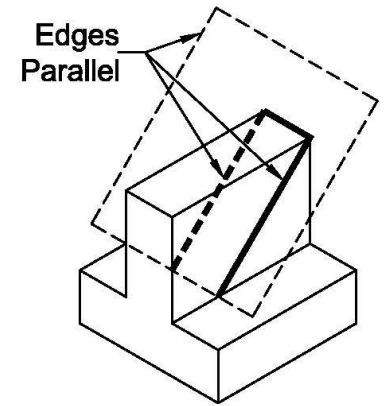
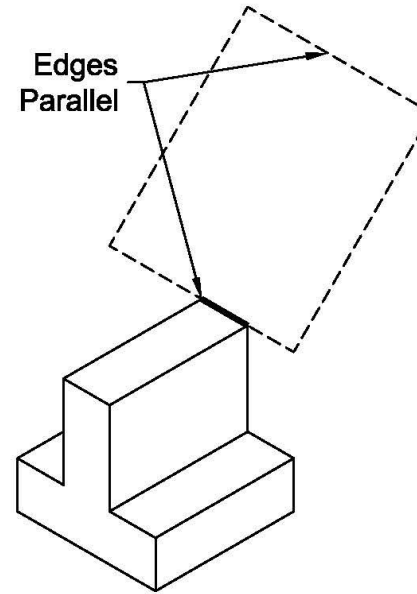


Possible Cross Sections

# Cutting Planes

As a plane cuts an object, the boundary edges on the cross section that results will be parallel to the edges of the cutting plane itself

- Rotate the plane into position to view it "straight on"



Cross Section Rotated to Viewing Plane

# Course Software...

- Work through the Cutting Planes and Cross Sections software module

# Cutting Planes and Cross Sections Homework ...

- Complete the following pages in Module 10
- cp/cs-
  - 1 - 2
  - 5 - 6
  - 9 - 10
  - 13 - 16
  - Hint: for pages 14 and 16 all problems have at least two correct answers